Chapter 11: Game Theory & Competitive Strategy

Game theory helps to understand situations in which decision-makers *strategically interact*.

A game in the everyday sense is: “A competitive activity ... in which players contend with each other according to a set of rules.”

Poker, chess are examples of a game but the scope of game theory goes much beyond: studies economic, political and biological phenomena:
- firms competing for business
- political candidates competing for votes
- animals fighting over a prey
- bidders competing in an auction
- role of threats and punishment in LT relationships

The major developments of game theory began in the 1920s with the work of 2 mathematicians: Émile Borel (1871-1956) and John von Neumann (1903-1957).

A decisive event is the publication in 1944 of the *Theory of Games and Economic Behavior* by von Neumann and Oskar Morgenstern.

In the early 1950s, another mathematician, John F. Nash developed a key concept of equilibrium (Nash equilibrium) and initiated the study of bargaining from a game-theoretic point of view. Since then, the tools of game theory have been applied in many different contexts, economics, political science, evolutionary biology. The 1994 Nobel Prize in Economic Sciences was awarded to 3 main contributors to the discipline, John Harsanyi, John Nash and Reinhard Selten.

Beyond their first names, Nash and von Neumann have several points in common.
- Both were truly geniuses: John von Neumann, Hungarian, had mastered calculus by the age of 8 and published his first scientific paper in 1922... at the age of 19. John Nash began his graduate work in 1948 at Princeton (one of his letters of recommendation was a single sentence: “This man is a genius”); he completed his PhD at the age of 22... his thesis was just 28 pages long!
- Both were involved in applied work and connected to Princeton University. In 1943, von Neumann became a consultant to the Manhattan Project which was developing an atomic bomb. Stayed at Princeton until 1954 when he was appointed to the U.S. Atomic Energy Commission.

To learn more about the life of John Nash, you can read *A Beautiful Mind* written by Sylvia Nasar or watch the movie which has the same title.

A **strategic game** is a model of interacting decision-makers which consists of:

- a set of players
- a set of actions for each player
- preferences over the set of action profiles: the payoffs

A **strategy** is a plan of action or a rule for playing the game; the rule specifies an action for every circumstance that might occur.

**Example 1:** “I’ll keep my price high so long as my competitors keep their price high; if one of them drops his prices, I’ll drop mine too”.

**Example 2:** “I will first make a serious bid of $2,000 to convince the other bidders that I am determined to win the auction; if the price goes above $5,000, I will however drop out.”
What is important is that players' payoffs do not only depend on their own strategy choice but also on the choices of the other players.

Key assumptions:

1. Players are selfish: all they care about is their own material payoffs.
2. Players are rational: they are taking the best action they can according to their preferences; they are acting to maximize their payoffs.
3. Rationality is common knowledge: “I am rational. I believe you are rational. I believe you believe I am rational. I also believe you believe that I believe you are rational...”
4. The structure of the game is common knowledge (games of perfect information): players know the set of other players, their own actions, the payoffs associated to each action profile.

Cooperative versus Noncooperative Games

Cooperative Game: game where players can negotiate binding contracts that allow them to plan joint strategies.

Example 1: Bargaining between a buyer and a seller over the price of pie. If the seller’s cost to producing the pie is $5 and the buyer is willing to buy it for $10, then a cooperative solution to the game is possible: any price between $5 + \varepsilon$ and $10 - \varepsilon$ maximizes total surplus and makes both parties better off. ⇒ constant sum game: no matter the selling price, total surplus (=sum of consumer surplus and producer surplus) is the same.

Example 2: Two firms negotiating a joint investment for a new technology; a binding contract specifies how much each firm invests and how the joint profits will be divided between the two firms. ⇒ nonconstant sum game: total profits of the joint venture will depend on the outcome of the negotiations i.e. on the resources invested by each firm.

Noncooperative Game: Game in which negotiation and enforcement of binding contracts are not possible.

Example 1: Cournot Duopoly game. Each firm takes the behavior of the other firm as given when independently choosing its quantity.

Example 2: Auction game. Each bidder chooses an optimal bidding strategy taking the behavior of other bidders as given.

Simultaneous-move games versus sequential games:

- In a simultaneous game, each player has to pick a strategy without being able to observe the strategy taken by his opponents; decisions are taken simultaneously.
- In a sequential game, players take their decision in sequence, after having observed the action taken by their predecessors.

The Prisoner’s Dilemma: Dominant Strategies

Probably the most studied game in economics, both from a theory and an empirical point of view.
Two prisoners have been accused of collaborating in a crime. They are in separate jail cells and they cannot communicate. They are both asked to confess. If both confess, they receive a prison term of 5 years. If neither confesses, prosecution becomes harder and they only get 2 years of prison each. However, if only one confesses, he receives one year of prison while the other receives 10 years. If you were a prisoner, what would you do?

<table>
<thead>
<tr>
<th>player A \ player B</th>
<th>Confess</th>
<th>Don’t Confess</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confess</td>
<td>-5, -5</td>
<td>-1, -10</td>
</tr>
<tr>
<td>Don’t Confess</td>
<td>-10, -1</td>
<td>-2, -2</td>
</tr>
</tbody>
</table>

Observe that if player A confesses, it is optimal for player B to confess (he gets -5 instead of -10); if instead player A doesn’t confess, it is again optimal for player B to confess (he gets -1 instead of -2). Therefore, confessing is optimal for player B no matter what player A does i.e. player B has a dominant strategy. Since the game is symmetric, confessing is also a dominant strategy for player A.

**Dominant strategy:** strategy which is optimal no matter what the other player does. We say that Confess is a strictly dominant strategy and Don’t Confess is a strictly dominated strategy.

Since, no player can do better than confessing given the other player’s strategy, the optimal strategy profile is (Confess, Confess) and each player gets −5. We say that (Confess, Confess) is an equilibrium in dominant strategies.

**Equilibrium in dominant strategies:** outcome of the game in which each player is doing the best he can regardless of what his opponent is doing.

Clearly, (Confess, Confess) is Pareto-dominated by (Don’t Confess, Don’t Confess). However players cannot sign an agreement to play this strategy profile and each player would have an incentive to deviate if the other doesn’t confess.

Experimental Evidence on the Prisoner’s Dilemma: Cooper, DeJong, Forsythe and Ross [1996]: subjects are recruited to play a PD in the lab. Each subject plays 20 games with different opponents. They find that 78% of subjects choose Confess in the last 10 rounds of play. However, when communication is allowed, the incidence of Confess tends to be lower; under some communication protocols, the incidence of Confess drops under 30%.

Sometimes, games can be solved by removing strictly dominated actions in an iterative process.

An action is strictly dominated if whatever the other players do, some other action is better.

<table>
<thead>
<tr>
<th>A \ B</th>
<th>Left</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>1, 0</td>
<td>3, 2</td>
</tr>
<tr>
<td>Bottom</td>
<td>2, 1</td>
<td>0, 2</td>
</tr>
</tbody>
</table>

Here Left is strictly dominated by Right for player B. On the other hand, player A has no strictly dominated strategy.
Consider now the following example and notice that at each stage, one strategy can be removed. We obtain a single outcome (Bottom, Center).

\[
\begin{array}{ccc}
\text{A} \setminus \text{B} & \text{Left} & \text{Center} & \text{Right} \\
\hline
\text{Top} & 0,2 & 3,1 & 2,3 \\
\text{Middle} & 1,4 & 2,1 & 4,0 \\
\text{Bottom} & 2,1 & 4,4 & 3,2 \\
\end{array}
\]

In some other games, the optimal action of a given player depends on what the other is doing. Consider this other example:

\[
\begin{array}{ccc}
\text{Jane} \setminus \text{Jim} & \text{Boxing} & \text{Opera} \\
\hline
\text{Boxing} & 2,1 & 0,0 \\
\text{Opera} & 0,0 & 1,2 \\
\end{array}
\]

What is the optimal strategy of Jane? Of Jim? Notice that if Jane thinks that Jim will go to the opera (boxing), it is optimal for Jane to go to the opera (boxing).

What is an equilibrium in this context? Remember that an equilibrium is a stable situation in which no one can do better by taking another decision.

In the present context, we will look for Nash equilibria.

We say that a strategy profile is a **Nash Equilibrium** if each player is doing the best he can given what the other is doing.

- Is (Boxing, Opera) an equilibrium? No, given Jim chooses Opera, Jane should deviate to Opera and given Jane chooses Boxing, Jim should deviate to Boxing. Similarly for (Opera, Boxing).

- Is (Boxing, Boxing) an equilibrium? Yes, if any player deviates given the other chose Boxing, he will be worse off. Similarly for (Opera, Opera).

- Therefore there are two Nash equilibria in pure strategies: (Boxing, Boxing) and (Opera, Opera).

Is Nash equilibrium the appropriate concept?

- doesn’t tell us which outcome obtains in the previous game.
- relies heavily on individual rationality: what if your opponent trembled from his optimal strategy?
- in this case, it might seem more reasonable to play what we call a maximin strategy: strategy that maximizes the minimum gain that can be earned.

Example:

\[
\begin{array}{cc}
\text{Don't invest} & \text{Invest} \\
\hline
\text{Don't Invest} & 0,0 & -10,10 \\
\text{Invest} & -100,0 & 20,10 \\
\end{array}
\]
- Investing is a dominant strategy for firm 2.
- (Invest, Invest) is the only Nash equilibrium of the game.
- However, what if firm 2 trembled and played Don’t invest? Then firm 1 would get -100.
- If firm 1 wanted to be cautious, it would choose Don’t invest because the worst that would happen in this case is to lose 10.

In some games, certain Nash equilibria simply seem unreasonable. Consider the following entry deterrence game:

<table>
<thead>
<tr>
<th>incumbent \ entrant</th>
<th>Enter</th>
<th>Stay Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accommodate</td>
<td>100, 20</td>
<td>200,0</td>
</tr>
<tr>
<td>Fight</td>
<td>70, -10</td>
<td>200,0</td>
</tr>
</tbody>
</table>

- The game has two Nash equilibria: (Accommodate, Enter) and (Fight, Stay Out).
- How to interpret (Fight, Stay Out)? This outcome doesn’t correspond to a robust steady state of an extensive game.
- Note indeed that this game has a dynamic structure. Since Fight is not a credible threat, (Accommodate, Enter) seems to be the only reasonable outcome.

A short introduction to auction theory

Auction market: market in which products are bought and sold through formal bidding processes.

They are widespread:
- U.S. Treasury running an auction to sell Treasury bills
- International Olympic Committee auctions off television rights.
- Ebay

Particularly valuable for items that do not have established market prices. Example: artwork

Auction Formats:

1. **English auction** (oral ascending): The seller solicits progressively higher bids. The current highest bid is known to bidders and the auction stops when no bidder is willing to surpass this highest bid. The item is sold to the highest bidder at a price equal to the amount he bid.

2. **Dutch auction** (oral descending): The seller begins by offering the item at a high price. If no potential buyer is willing to pay that price, the seller then decreases his price until someone accepts. The first buyer to accept an offered price buys the item at that price.

3. **First-Price sealed bid auction**: All bids are made simultaneously in sealed envelopes. The winner is the bidder who submitted the highest bid. The price he pays corresponds to his own bid.

4. **Second-Price sealed bid auction**: All bids are made simultaneously in sealed envelopes. The winner is the bidder who submitted the highest bid. The price he pays corresponds to the second highest bid.
Valuation and Information:

1. **Private value auction**: auction in which each bidder knows his or her individual valuation of the object which is auctioned off. Valuations differ from bidder to bidder.

   Example: Baseball cards.

2. **Common value auction**: auction in which the item has the same value to all bidders but the bidders do not exactly know its value and their estimates vary.

   Example: Offshore oil reserve; value of the reserve = (price of oil - extraction cost) x amount of oil in the reserve.

   Auction to purchase a large jar of pennies. This is a common value auction: the value of the jar is the same for all bidders but each bidder has a different estimate of its value.

Suppose you have formed your own estimate of the value of the jar. What is your optimal bidding strategy?

You might feel tempted to just bid up to your own estimate and no higher. But wait, no bidder knows exactly what’s the value of the jar and all estimates are subject to some error. If each bidder were to bid his own estimate, there is a high chance that the winner is the one who made the largest positive error i.e. the one who overestimated the most the value of the jar. This is what we call the **Winner’s Curse**.

To account for this curse, you should reduce your maximum bid below the value of your estimate by an amount which corresponds to the expected error made by the winner. The more precise is your estimate, the smaller is the adjustment you will have to make. How would you know if your estimate is precise or not? If you see a lot of variance around you in the estimates provided by the other bidders, you should suspect that your estimate might be quite imprecise.