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Creating competition out of thin air: An experimental study of right-to-choose auctions

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Abstract

This paper presents an experimental study of a mechanism that is commonly used to sell multiple heterogeneous goods. The novel feature of this procedure is that instead of selling each good in a separate auction, the seller executes a single auction in which buyers, who may be interested in completely different goods, compete for the right to choose a good. We provide experimental evidence that a Right-to-Choose (RTC) auction can generate more revenue than the theoretically optimal auction. Moreover, in contrast to the "optimal" auction, the RTC auction is approximately efficient in the sense that the surplus it generates is close to the maximal one. Furthermore, a seller who would like to retain some of his goods can generate more revenue with a restricted RTC auction in which not all rights-to-choose are sold, than with the theoretically optimal auction.

JEL classification: C91; D44

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1. Introduction

This paper presents an experimental study of a mechanism that is commonly used to sell multiple heterogeneous goods. The novel feature of this procedure is that instead of selling each good in a separate auction, the seller executes a single auction in which buyers, who may be

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interested in completely different goods, compete for the right to choose a good. Right-to-choose auctions (or RTC for short) are commonly used to sell real-estate, antiques, jewelry and most recently, customized telephone numbers.[1] The conventional wisdom regarding this form of auction is best summarized by the following quote from the *Maine Antique Digest* (which refers to RTC auctions as "Choice Selling"),

> "Choice selling sometimes has the ability to generate higher selling prices than straight item-by-item offerings. Auctioneers often don't know who has the most interest in what piece and how much that bidder might be willing to pay to own it. The auctioneer wants to push that bidder as far as he can (and he should), so long as it's done legally and ethically. One way to do this is to force bidders to compete who would not otherwise do so, and choice selling does that."[2]

From a theoretical standpoint, there is no basis for this conventional wisdom as this auction format is revenue equivalent to an auction in which each good is sold separately (with no reserve price). However, we provide experimental evidence that supports the conventional wisdom by demonstrating that in the lab, this form of auction can generate more revenue than the theoretically optimal auction. Furthermore, a seller who would like to retain some of his goods can generate approximately the same revenue with a restricted RTC auction in which some rights-to-choose are not sold.

For expositional purposes we illustrate the subject of our study with the following simple example. Consider a record collector who has been collecting records over his lifetime. Suppose the collector decides to sell three albums of three distinct music styles. He chooses to sell one opera album (*Stravinsky's "Oedipus-Rex"*), one punk-rock album (*The Sex Pistols*) and one pop album from the 1980s (*Duran Duran*). On the day of the sale six people show up to bid. However, because the three albums are so different the people who show up have very specific tastes: two value only the opera album and have no use for the others, another pair of buyers only value the punk album and the remaining two are only interested in the 1980s album.

The record collector has to decide how to auction the records. One option is to hold three separate auctions and run them either sequentially (offering one record at a time as they would do in Sotheby's) or simultaneously (allowing bidders to bid simultaneously on any record they want until the auction ends). We call such auctions Good-by-Good auctions, or GBG for short.

An alternative is to combine the 'thin' markets for each of the goods into one 'thick' market by transforming the three distinct goods into three units of a new good called a "right-to-choose." A RTC auction would consist of three phases. In the first phase, all six bidders submit sealed bids. The highest bidder wins the right to choose one of the three goods, and he pays the second-highest bid. The other five bidders are then told which good was taken, and the bidder, who wanted the same good as the winner, exits the auction. The remaining four bidders enter the second phase of the auction, where again bidders submit sealed bids, the highest bidder wins the right to choose one of the two remaining goods, and then both he and another bidder who wanted the same good as the winner, exit the auction. At the third and final phase, two bidders compete in what is essentially a standard second-price auction.

[1] RTC auctions are also known as "Bidders' Choice" auctions. By Googling either terms one can find many sites that offer this auction format.

[2] This citation is taken from a reply by the Digest's legal correspondent to a letter by a reader, who complained about the frequent use of this auction format. See: http://www.maineantiquedigest.com/articles/feb04/ethi0204.htm.

Notice that this auction forces buyers of completely different goods to compete for the same good, namely the 'right-to-choose.' This can lead the highest bidder for one album to pay the bid of the highest bidder for another album. While this feature seems appealing, the RTC auction is actually revenue equivalent to the sequential or simultaneous GBG auctions described above: in the symmetric equilibrium, both types of auctions lead to the same efficient allocation of albums.

We conduct an experimental analysis of RTC auctions in a simple environment that captures the basic ingredients of our record collector example. A monopolist owns K unrelated goods. For each type of good, there are n risk-neutral buyers who value only that good. In our experimental design, $K = 4$ and $n = 2$. Buyers' valuations are purely private and are drawn independently from a uniform distribution on [0, 1]. All this information is common knowledge among all players.

The simplicity of the model underlying our experiment has several appealing features. First, it allows us to isolate how the mere change of auction format (from GBG to RTC) affects bidding behavior. We achieve this by controlling for the information that bidders have in each type of auction (in both formats, each bidder knows the number of buyers who want the same good as he does), and by focusing on a preference structure where there is least reason to compete with buyers who demand other goods. Second, the optimal mechanism is easy to derive and has a simple structure: it consists of K separate second-price auctions with a reserve price of $\frac{1}{2}$. Finally, our model lends itself to a simple experimental design, which reduces the concern for misunderstanding on part of the subjects.

We interpret our set-up as a stylized model of situations in which heterogeneous goods of comparable value would be sold in separate independent auctions. For example, in art auctions such as those held by Sotheby's or Christie's—the set of buyers who bid for one piece of art (say, a painting) is often distinct from the buyers who bid for another piece of art (say, a sculpture). In online auction sites, such as e-Bay, there are many instances in which a single seller offers different types of the same product—DVDs, CDs, books, different brands of electrical appliances—in separate auctions with mutually exclusive sets of buyers. Our model applies in just such situations where goods exist of comparable value but for which the set of buyers do not overlap.

Our experimental findings lend support to the idea that RTC creates "competition out of thin air" in the sense that buyers of different goods compete against each other. Not only does the RTC auction generate significantly more revenue than a GBG auction, *it generates more revenue than the theoretical optimal auction*.[3] Moreover, in contrast to the "optimal" auction, the RTC auction is approximately efficient in the sense that the surplus it generates is close to the maximal one.

These findings raise the question of whether forcing buyers of one good to *really* compete with the buyers of another good will further enhance competition. We investigate two ways in which this can be achieved: restricting quantity and withholding information. Our experimental findings demonstrate that quantity restriction leads to more aggressive bidding behavior. Specifically, when quantity is restricted by one unit, more revenue is raised in each phase of the restricted RTC auction than in the corresponding phase of the unrestricted auction. However, given the high revenue of the RTC auction when all goods are sold, revenues are approximately the same in both auction formats. Still, a seller who faces unexpected liquidity constraints and who is forced to sell goods against her will, may prefer a RTC format with quantity restriction because it allows her to retain one of the goods without losing revenue.

[3] The higher revenue in the RTC auction does *not* diminish over repetitions of the game.

Next we look at a RTC auction with no information (NIRTC for short). In this variant of the RTC format, at the end of each phase, bidders are not informed of the good taken in the previous round, and are not given the opportunity to exit the auction. Winning bidders whose good was taken in a previous phase are randomly assigned one of the remaining goods. This is essentially a sequential variant of what is called a "pooled auction" where each bidder submits a single sealed-bid and the rank-order of his bid determines his place in the queue for choosing among the available goods.[4] We find that this auction format also outperforms the GBG auction with optimal reserve prices, but fails to raise more revenue than the unrestricted RTC auction.

Finally, we check the combined effect of withholding information *and* restricting quantity. Our data suggests that withholding information *and* restricting quantity is no better than conducting a RTC auction with only one of these features.

The fact that our RTC auctions outperform the optimal auction in our experimental environment means that the behavior observed differs from that predicted by the theory. Since the observed behavior in the optimal auction closely follows the theoretical prescription, our results are driven by the fact that the RTC design induces a bidding behavior that departs from the theory. We show that the bidding data cannot be reconciled by introducing risk aversion in the model. In agreement with the data, risk aversion raises the bids and revenue in the RTC auctions. In contrast to the data, however, risk aversion produces cautious bids and lower revenue in the NIRTC auction compared to the risk neutral case.

To explain our experimental findings, we propose a parsimonious departure from the standard theoretical model that captures the conventional wisdom that RTC auctions artificially enhance competition between buyers who would otherwise refrain from competing. More specifically, we hypothesize that bidders behave *as if* the number of buyers who are after their good is higher than it actually is. We provide intuition motivating this idea as well as evidence that this model of behavior provides a good fit for bidding patterns in the data.

The paper is organized as follows. We begin with a discussion of related literature in Section 2. In Section 3 we present the theoretical predictions that we will test using the experimental design described in Section 4. Section 5 summarizes our experimental findings and Section 6 concludes.

2. Related literature

As mentioned in the Introduction, auction formats, where instead of winning a specific object, bidders win the right-to-choose any of the yet unsold objects, are often used to sell real-estate such as condominiums and land parcels. Most of these auctions either use a sequential design similar to the one we use here (see Ashenfelter and Genesove, 1992), or they follow the simultaneous "pooled auction" format. Menezes and Monteiro (1998) derive the optimal bidding strategies for risk-neutral bidders in a pooled auction and show that in the homogeneous private-values case, this format is revenue-equivalent to a sequential auction of multiple items.

In an experimental study, Salmon and Iachini (in press) compare the pooled auction to the simultaneous ascending auction and find that pooled auctions raise higher revenues than the simultaneous ascending auction. The pooled auction has the disadvantage that bidders frequently experience losses when they are forced to buy their less preferred goods at high prices. Salmon and Iachini report that the overbidding in their pooled auctions can neither be explained by loss aversion nor by risk aversion. Instead, they propose that an "attentional bias" explains the over-

[4] See our discussion of Burguet (2005) in the next section.

bidding observed in the pooled auction. According to this bias, bidders tend to focus on their top items and tend to ignore the possibility of winning the less preferred goods.

Our explanation of overbidding in the RTC auctions is related to the explanation offered by Salmon and Iachini in the sense that both explanations imply that bidders distort objective probabilities. The attentional bias proposed by Salmon and Iachini assumes that subjects underweight the possibility of winning their less preferred items, which these authors show can account for overbidding in pooled auctions as well as in all-pay auctions. We, instead, propose that subjects fail to realize that, despite the possibly large number of bidders in an RTC auction, they need only concentrate on beating those bidders who value the same good they do. If they fail to realize this, subjects will overestimate the number of bidders they are competing with, and consequently, distort the probability that their good will be taken in each phase.

Thus, we propose an explanation that is nested in the general class of probability (mis-)weighting models. Recently, this approach has been employed to successfully model subjects' behavior in auctions. A number of authors (e.g., Isaac and Walker, 1985; Neugebauer and Selten, 2003; Ockenfels and Selten, 2005) have shown that bids in first-price auctions are closer to equilibrium when subjects observe their opponents' highest bid in each period. While these papers did not explicitly elicit beliefs, their results suggest that once subjects have better possibilities to learn about others' behavior, their beliefs become more precise and overbidding becomes less prominent. Further support is provided by Armantier and Treich (2005), who elicit subjects' beliefs in first-price private value auctions, and show that subjects misperceive the probability of winning. According to their findings, subjects tend to underestimate their probability of winning, which appears to be a significant factor explaining overbidding in first-price private value auctions.

Burguet (2005) studies a model with two substitute goods and shows that the RTC auction in which all goods are sold is efficient but not optimal. However, the optimal mechanism turns out to be quite complex requiring information, which is not readily available to the seller. Burguet proposes a detail-free way of introducing inefficiency into the RTC auction, which improves the performance of the original auction: the seller should not reveal to the bidders which item was chosen by the winner of the previous phase. Clearly, in such an auction a winning bidder may wind up paying a price higher than his value for the good he wins. Consequently, winning bidders may wish to opt out when their willingness to pay for each of the available goods is smaller than the price they need to pay.[5] We propose quantity restriction as an alternative means for introducing inefficiency into the RTC auction in a detail-free manner. Furthermore, we present experimental evidence that the RTC auction in which bidders are informed of the good selected each phase, generates more revenue than when this information is withheld.

Goeree et al. (2004) introduce risk-averse bidders into Burguet's (2005) model. They show that in this case, RTC auctions in which *all goods are sold* raise more revenue than standard simultaneous or sequential ascending auctions. These authors also test their model in the laboratory, and provide evidence in support of their theoretical result. These findings suggest that the performance of the RTC auction may be robust to the buyers' degree of risk aversion. This contrasts with the theoretical optimal auction, which is sensitive to risk aversion. Goeree et al. (2004) do not consider the possibilities of quantity restriction or withholding information about previous winners, and they do not compare their results with the optimal auction in their set-up. Notice that risk aversion explains the experimental results observed in RTC auctions but not the experimental results observed in pooled auctions.

[5] In fact, under California law, winning bidders in pooled auctions have the right to opt out.

3. RTC auctions in theory

In this section we present the environment we shall focus on and derive the theoretical predictions for this environment. These predictions will serve as a benchmark for comparing our experimental findings. Most of the results in this section are derived using standard tools from the analysis of sequential auctions (e.g., Milgrom and Weber, 2000).

We consider a seller with K heterogeneous goods who faces the following demand structure. For each of the K goods there are exactly n risk neutral buyers. A randomly chosen buyer is equally likely to demand any of the K goods. Each buyer has a private value for only one of the K goods (the 'preferred' good), and has zero value for all other goods. All buyers independently draw the value for their preferred good from a uniform distribution on [0, 1].[6] All of the results that we report in this section are in reference to this model. We relegate all proofs to the appendix.

There are several selling procedures that accommodate the above demand structure. The benchmark procedure consists of holding K separate second-price auctions, one for each good. We call this procedure a *good-by-good* (GBG) auction and denote it by $GBG(K, n)$, where K stands for the number of goods and n for the number of buyers per good.

An alternative selling procedure is a *right-to-choose* (RTC) auction, which proceeds as follows. There are $K - q \leqslant K$ phases, where q (which stands for "quantity-restriction") is a nonnegative integer, which is smaller than K. In the first phase, all nK bidders bid for the right to choose among the K available goods. The highest bidder in phase 1 wins the right to choose one of the K goods and pays the bid of the second highest bidder. At the end of this phase bidders are told which good was selected by the winner (bidders are not informed of the price paid by the winner). Bidders are then given the option to either exit the auction or stay and move to the next phase. Clearly, all of the $n - 1$ buyers who value the same good as the winner will at this point drop out of the auction. The remaining bidders participate in phase two, which is essentially the same as phase 1: bidders simultaneously submit bids, the highest bidder wins the right to choose one of the remaining goods, the winner pays the second highest bid and chooses a good, and all other bidders are informed of the good that was chosen. This continues until $K - q$ rights offered for sale are sold. A sequential right-to-choose auction with K goods, $K - q$ phases and n bidders per good is denoted $RTC(K, K - q, n)$.[7] We choose the second-price rule to help make the analysis more tractable since this pricing rule simplifies the derivation of the equilibrium strategies.

A strategy in an $RTC(K, K - q, n)$ is a collection of $K - q$ functions, one for each phase, where each function maps a bidder's value into a bid for the corresponding phase of the auction. The second-price auction in each phase can be thought of as being an ascending-clock English

[6] All of our results can easily be generalized for a uniform distribution on $[\underline{v}, \bar{v}]$. We assume that for each good the seller has a value equal $\underline{v}$. The possibility to reduce quantity in a right-to-choose auction becomes even more attractive in situations where the seller is forced to sell the goods as a result of financial trouble while she has higher values for the goods than the lower end of the support.

[7] In principle, an RTC auction could also be run simultaneously where bidders submit $K - q$ bids (one bid for each phase of the auction) all at once in addition to declaring which good they are interested in buying. We chose a sequential design because it highlights the trade-off between having a higher chance of obtaining a good now and paying a lower price in the future.

Auction in which the price rises until the pen-ultimate bidder drops out and the remaining bidder wins at the last drop-out price (to maintain strategic equivalence with the second-price format, we assume that the auctioneer does not reveal drop-out prices of bidders who leave the auction). Hence, each bidder in the auction must determine a drop-out price for each phase in which he or she is still active. For a bidder with value v we denote this phase-k drop-out price by $b_k(v)$, where $k=1$ is the initial phase and $k=K-q$ is the final phase.

We focus on the symmetric perfect Bayesian Nash equilibrium (SPBE) in which all bidders use the same monotonic bid function in each phase. Let N denote the total number of bidders (i.e., $N=nK$) and let N_k denote the number of active bidders in phase k of the auction (i.e., $N_k=n(K-k+1)$).

Proposition 1. *The RTC(K,K,n) has an SPBE with the property that in each phase $1\leqslant k\leqslant K$ every bidder uses the linear bid function $b_k(v)=\frac{(N_k-1)-(K-k)}{N_k-1}v$.*

To understand the intuition for this result let $K=2$ and $n=2$. In the second and final phase there are two bidders competing in a standard second-price auction, hence, $b_2(v)=v$. To derive $b_1(v)$ let us compare the payoff from bidding according to v and bidding just below (or above) that value. We can make this comparison, conditional on the event that the highest valuation of the other bidders is also v, since this is the only event in which these two bids result in different outcomes. Thus, bidding according to one's true value in phase 1 leads to an expected payoff of $v-b_1(v)$. Bidding slightly below one's value would result in losing the first phase and, provided the good is still available (the probability of which is $\frac{2}{3}$), winning the second phase (where the expected price is $\frac{1}{2}v$). Hence, the expected payoff from bidding below one's value is $\frac{2}{3}(v-\frac{1}{2}v)$. The equilibrium bid, $b_1(v)$, should make the bidder indifferent between bidding according to v and bidding slightly below, hence, $b_1(v)=\frac{2}{3}v$.

If buyers bid according to Proposition 1, then combining K markets together and letting all bidders compete against each other for "rights-to-choose," does not lead to a higher expected revenue in equilibrium.

Proposition 2. *The expected revenue in the SPBE of an RTC(K,K,n) is equal to the expected revenue obtained in a GBG(K,n) where bidders use weakly dominating strategies.*

By introducing some inefficiency into the auction, one can break the revenue equivalence of RTC and GBG. This can be achieved by taking advantage of the fact that the seller in a RTC auction is simply a monopolist with a fixed supply of K rights-to-choose. When facing an inelastic known demand, a monopolist with a fixed supply would maximize revenue by selling less units than what he actually has. The question is, could a monopolist in our setting also increase his revenue by restricting quantity? We address this question for the case in which the monopolist reduces quantity by one unit. Applying the same intuition of Proposition 1 to RTC with quantity restriction yields the following.

Proposition 3. *The RTC$(K,K-1,n)$ has an SPBE with the property that in each phase $1\leqslant k\leqslant K-1$ every bidder uses the linear bid function*

$$b_k(v)=\frac{N_k-(K-k)}{N_k-1}v. \tag{1}$$

Note that quantity restriction enhances the competition between bidders, as evident from the fact that the bidding coefficient in phase k of $RTC(K, K-1, n)$ is higher by $\frac{1}{N_k-1}$ than the corresponding bid in the $RTC(K, K, n)$. Since there is one less phase in the restricted RTC auction, it is not immediate that the rise in bids would generate higher revenues. However, for any n there is K large enough such that selling all but one good raises more revenue. In particular, for the setting we use in our experiments, where $n = 2$, quantity restriction raises revenue for any $K > 2$.

Proposition 4. *For every n there exists a finite $K(n)$ such that for all $K \geqslant K(n)$, restricting quantity by one unit raises the expected revenue in the SPBE of the RTC auction. In particular, for $n = 2$ we can set $K(2) = 3$.*

As mentioned in the Introduction, an alternative way to introduce inefficiency into the RTC auction is to withhold information from bidders by making the following change in the original RTC design described above: at the end of each phase, *bidders are not informed of which good was taken by the winner of that phase*. That is, a bidder who wins a phase, first pays the second highest bid, and then gets to choose one of the available goods. When a winner of some phase does not find the good he values, he picks one of the remaining goods at random. We interpret this assumption as saying that a bidder does not want to leave an auction empty handed (especially if he wins and pays).[8] We call this auction format an RTC auction with no information, or NIRTC for short. An NIRTC with K goods, $K - q$ phases and n bidders per good is denoted $NIRTC(K, K-q, n)$. The SPBE bids for this auction, with and without quantity restriction, can be derived using the same intuition behind Proposition 1. In the NIRTC auction, the number of bidders in phase k is defined as: $N_k = nK - k + 1$.

Proposition 5. *For all $q < K$, the $NIRTC(K, K-q, n)$ auction has a SPBE in which bidders use a linear bid function in every round. The bidding coefficients $(\beta_k)_{k=1}^{K-q}$ are given by*

$$\beta_{K-q} = \frac{n(q+1)}{(n-1)K + q + 1}$$

and the unique solution to the following system of difference equations

$$\frac{N(n-1)}{N_k N_{k+1}} = \beta_k - \frac{N_{k+1} - 1}{N_{k+1}} \beta_{k+1}$$

where $k = 1, \ldots, K - q - 1$.

Similar to quantity restriction, withholding information also leads buyers of different goods to compete against each other since the winner of each phase takes a good, whether or not he is interested in that good. In our experimental design, we focus on an environment where the market for each good is "thin" in the sense that only two buyers demand each good ($n = 2$). In this environment, the enhanced competition among buyers leads to higher expected revenues.

Proposition 6. *The expected revenue in the SPBE of the $NIRTC(K, K, 2)$ is higher than that of the $RTC(K, K, 2)$.*

[8] We could formalize this interpretation by assuming that for each bidder there is only one good from which his value is drawn from a uniform distribution on $[\varepsilon, 1]$, while for the other $K - 1$ goods his value is $0 < \mu < \varepsilon$, and where ε is arbitrarily close to zero.

In light of Proposition 6, it is natural to ask whether quantity restriction could help raise further the expected revenue in an NIRTC.

Proposition 7. *Quantity restriction lowers the expected revenue in the NIRTC auction: for all $q < K$, the expected revenue in the SPBE of NIRTC$(K, K - q - 1, n)$ is less than the expected revenue in the SPBE of NIRTC$(K, K - q, n)$.*

While quantity restriction raises revenue in a standard RTC, it reduces revenue in an NIRTC. Roughly speaking, this follows from the fact that by eliminating the Kth phase in an NIRTC auction we lose the bid of the $K + 1$ highest bidder. In contrast, by the time we get to the last phase in an RTC auction, there is a high probability that the $K + 1$ highest bidder has already left the auction, hence eliminating this phase leads to a smaller loss.

We conclude this section by noting that the optimal selling procedure in our framework has a very simple structure.

Proposition 8. *Expected revenue is maximized by a GBG(K, n) auction with a reserve price of $\frac{1}{2}$.*

To understand the intuition for this result, note that our assumptions imply that the payoff type of each bidder is uni-dimensional: it can be summarized by the value he assigns to his preferred good. Because bidders draw this value independently from the same distribution, there is no loss of generality from finding the optimal mechanism for a single market and conducting K separate replicas of this mechanism. A straightforward application of Riley and Samuelson (1981) yields that an optimal auction in the market for good k is a second-price auction with a reserve price of $\frac{1}{2}$.

4. Experimental design and procedures

4.1. Design

The experiment we ran consisted of six treatments. Each treatment corresponded to one of the six auction formats investigated in the previous sections: a standard RTC auction in which all goods are sold (*RTC*(K, K, n)), an RTC auction in which one good was not sold (*RTC*$(K, K - 1, n)$), an NIRTC with all goods sold (*NIRTC*(K, K, n)) and with one good not sold (*NIRTC*$(K, K - 1, n)$), a GBG auction where each good is sold using a second-price rule with *no* reserve price (*GBG*(K, n)) and a GBG auction with an optimal reserve price of 50 (*OGBG*(K, n)).

Since the tools to intensify competition are most relevant in thin markets, we set the number of bidders for each good equal to two ($n = 2$) in all treatments. We chose $K = 4$, so that in total eight subjects were competing for four goods. In some auctions we restricted the number of rights to be sold to $K - 1 = 3$, so our auctions consisted of *RTC*$(4, 4, 2)$, *RTC*$(4, 3, 2)$, *NIRTC*$(4, 4, 2)$, *NIRTC*$(4, 3, 2)$, *GBG*$(4, 2)$ and *OGBG*$(4, 2)$. The value for each subject was independently drawn from a uniform distribution over the support [0, 100]. The experiments were performed at CREED, the experimental economic laboratory of the University of Amsterdam, as well as the experimental laboratory at the Center or Experimental Social Science at New York University.

Table 1
Experimental design

Treatment	Number of groups	Subjects per group	Total
RTC(4, 4, 2)	8	8	64
RTC(4, 3, 2)	8	8	64
NIRTC(4, 4, 2)	8	8	64
NIRTC(4, 3, 2)	8	8	64
GBG(4, 2)	8	8	64
OGBG(4, 2)	8	8	64
Total			384

For each experiment subjects were recruited from the general undergraduate population of these respective schools. The experiment lasted about one hour and twenty minutes, except for the good-by-good auction which typically lasted about one hour. Subjects were paid a show up fee in each location and earned the remainder of their money according to how they did during the experiment. Motivation and understanding of the instructions were good and average earnings were $18.2 in Amsterdam and $15.8 in New York. No significant behavioral differences were found across locations so we pool all observations from both subjects populations.[9]

Each group of subjects performed one and only one type of auction and repeated the auction 16 times after participating in a practice round. There were eight groups performing each treatment (four in Amsterdam and four in New York) so the total number of subjects recruited was 384 (eight groups of eight subjects in six treatments). Four different sets of values were generated for the first four groups of subjects in each treatment. The same exact sets of values were also used for the second four groups. Hence, each set of randomly generated values was used twice in the experiment in each treatment (once in New York and once in Amsterdam). We did this to ensure that any revenue differences were attributable to differences in behavior rather than differences in the vectors of random variables generated. This also allowed us to make some controlled comparisons of behavior.

Our design is summarized in Table 1.

4.2. Procedures

In all six treatments subjects were seated in a computer lab in groups of 16 and separated into two sub-groups of eight subjects each. Subjects read the computerized instructions at their own pace. The instructions of the $RTC(4, 3, 2)$ auction are presented in Appendix B. Each group performed the same experiment, but once a subject was assigned to a group of eight, he or she remained in that group for the entire experiment. In all treatments each of the 16 periods began by each subject being shown the good he or she valued (either good A, B, C, or D) and the value of that good for the period. After this was presented on the screen the program asked them to bid. In all of the RTC auctions subjects were asked to bid in phases.

The RTC auctions proceeded as follows. In phase 1 subjects submitted their bid and, using the second price rule, the good was allocated to the highest bidder at the second highest price. If the winner selected the good of another subject, that subject was informed that his or her good was selected and was not allowed to bid in further phases of this period. The vector of submitted bids

[9] There was a small set of subjects who went bankrupt in New York. All observations occurring after these bankruptcies happened were dropped. This was the only difference in the behavior of subject pools noticed.

was not revealed to the bidders. The winner was the only one who knew the price at which he or she had bought the good. The bidders whose good was not won in the initial phase proceeded to phase 2. This phase, as well as those that followed it, proceeded in the same manner as phase 1.

In the NIRTC auctions, after each phase, no information was offered as to which good was chosen. All that subjects were told was that some good had been chosen. Bidding then continued as it did in the previous phase. If the good preferred by a winning bidder had been selected previously, the bidder was assigned a good at random (a good for which he or she has a zero value). At the end of the period the earnings of each subject was placed on the screen as was the cumulative earnings of the subject up until that period.

When the next period began, subjects were allocated to different goods at random. Hence, each subject was randomly paired with one other subject who valued the same good as he. A new independent value was presented to them and bidding proceeded in phases as before. In $RTC(4,4,2)$ and $NIRTC(4,4,2)$ there were four phases per period while in $RTC(4,3,2)$ and $NIRTC(4,3,2)$ there were three phases per period. Finally, in the $GBG(4,2)$ and the $OGBG(4,2)$ auctions, each period consisted of only one phase in which all subjects bid for the good they valued and faced one other subject who also valued that good.

Total earnings in the experiment consisted of the per-period earnings of subjects summed over all 16 periods. Subjects played for points which were converted into euros and dollars at the rate of 15 points per one dollar. Finally, to protect subjects from bankruptcy we gave each subject a 150 points at the beginning of the experiment and all losses during any period were subtracted from this amount.[10]

5. RTC auctions in the lab

From the analysis in Section 3 it follows that in the SPBE of the auction, subjects are expected to bid in each phase according to a linear bid function with the property that the ratio of bid to value increases from phase to phase in a RTC auction and decreases in a NIRTC auction. The equilibrium bid coefficients obtained for the parameters of our experimental design, are presented in Table 2.

Table 2
Equilibrium bid coefficients and expected revenues

Auction format	Phase 1	Phase 2	Phase 3	Phase 4	Expected revenue *theory*	Expected revenue *values*
$RTC(4,4,2)$	4/7	3/5	2/3	1	133.3	136.2
$RTC(4,3,2)$	5/7	4/5	1	NA	152.1	157.7
$NIRTC(4,4,2)$	1599/2205	214/315	44/75	2/5	152.1	155.2
$NIRTC(4,3,2)$	115/147	47/63	2/3	NA	147.6	149.4
$GBG(4,2)$	1	NA	NA	NA	133.3	140.1
$OGBG(4,2)$	1	NA	NA	NA	166.7	174.0

[10] Of the 384 subjects, nine went bankrupt and this occurred in six groups (i.e. in some groups several people went bankrupt). All bankruptcies except one occurred in the $NIRTC(3,4,2)$ and $NIRTC(4,4,2)$ experiments. The one other bankruptcy occurred in the $RTC(4,4,2)$. Six of the nine bankruptcies occurred after period 11 (four in period 14). To purge the impact of bankruptcy on the data we drop all observations for subjects in any group after a subject had gone bankrupt.

Table 2 also compares two notions of expected revenue. The first notion, displayed in the fifth column, computes the expectation with respect to *all possible realizations of values*. This is the expected revenue predicted by the theory. The second notion, displayed in the last column, computes the expectation with respect to the *values drawn in our experiment*. This is the revenue that is expected to be generated when subjects with the values generated in our experiment bid according to the equilibrium. While the first notion assumes that the law of large numbers is at work in the generation of values, the second notion accounts for the fact that set of values generated in the experiment is finite. Note that there is little qualitative or quantitative difference between the numbers in these two columns.

5.1. Revenue and efficiency

Before we analyze the bidding behavior of our subjects, we compare the performance of the auctions in terms of revenue and efficiency. Table 3 presents the mean revenues generated by our subjects in each phase and treatment along with their standard deviations. It also presents the revenues expected to be generated at the SPBE of our auctions along with the expected theoretical standard deviations.

A major finding in Table 3 is that the $RTC(4,4,2)$ and $RTC(4,3,2)$ raised substantially more revenue than predicted by the theory. Not only did the $RTC(4,4,2)$ outperform the $GBG(4,2)$, *it raised significantly more revenue than the theoretical optimal auction*, the $OGBG(4,2)$. The mean revenue of the $RTC(4,4,2)$ is 203.7, whereas the mean revenue of the $OGBG(4,2)$ is only 178.8. A Mann–Whitney test ran on the sample of revenues generated by these auctions indicates that this difference is significant at the 1% level.[11] The mean revenue of the GBG auction with no reserve price is only 145.1, which is significantly less than that raised by the optimal auction at the 1% level.

The data in Table 3 shows that quantity restriction leads to more aggressive bidding in each of the first three phases of the RTC auction. This is evident by noting that in each of these phases the revenue raised by the $RTC(4,3,2)$ is greater than that raised in the $RTC(4,4,2)$. Because

Table 3
Actual revenues

	Phase 1	Phase 2	Phase 3	Phase 4	Total
$RTC(4,4,2)$ actual	72.1 (14.4)	61.2 (15.8)	47.9 (18.2)	22.6 (15.1)	203.7 (51.1)
Nash	45.2 (7.3)	39.2 (9.6)	32.6 (12.4)	19.1 (13.3)	136.2 (33.7)
$RTC(4,3,2)$ actual	74.2 (14.3)	64.9 (15.9)	49.7 (18.8)	NA	188.7 (42.4)
Nash	56.4 (9.4)	52.5 (12.7)	48.7 (18.9)	NA	157.7 (35.5)
$NIRTC(4,4,2)$ actual	77.3 (14.8)	62.3 (15.5)	37.6 (16.4)	19.4 (12.6)	196.7 (46.7)
Nash	57.5 (9.7)	45.7 (10.3)	33.6 (9.8)	18.5 (7.0)	155.2 (32.1)
$NIRTC(4,3,2)$ actual	79.9 (14.3)	64.3 (16.3)	42.9 (18.3)	NA	187.1 (41.5)
Nash	62.1 (10.0)	49.5 (10.6)	37.8 (11.0)	NA	149.4 (27.6)
$GBG(4,2)$ actual	NA	NA	NA	NA	145.1 (55.2)
Nash	NA	NA	NA	NA	140.1 (52.9)
$OGBG(4,2)$ actual	NA	NA	NA	NA	178.8 (52.4)
Nash	NA	NA	NA	NA	174.0 (55.9)

Note: Standard deviations are listed in parentheses.

[11] All test results use independent average data per group as observations and groups with bankruptcies are not used. Unless indicated otherwise, Mann–Whitney tests are used.

bids in the *RTC*(4, 4, 2) were already higher than those predicted, the bid increment induced by quantity restriction was not sufficient to completely offset the fact that only three goods were sold. Indeed, the difference in revenues between the *RTC*(4, 4, 2) and the *RTC*(4, 3, 2) is not statistically significant.

According to Table 3 a restricted RTC generates significantly more revenue than a GBG auction with no reserve prices. In particular, the mean revenue of *RTC*(4, 3, 2) is 188.7, whereas the mean revenue of the *GBG*(4, 2) is only 145.1. According to a Mann–Whitney test, the difference between these two amounts is significant at the 1% level. Furthermore, the mean revenue of the *RTC*(4, 3, 2) is strictly higher than that of the *OGBG*(4, 2), though this difference is not statistically significant. This suggests that a seller, who does not have enough information to set optimal reserve prices, may benefit from conducting an RTC auction with quantity restriction.

Our experimental findings suggest that withholding information in an RTC auction may generate more revenue than that generated by a GBG auction with optimal reserve prices. The *NIRTC*(4, 4, 2) auction raised a significantly higher revenue than the *OGBG*(4, 2) auction (at the 5% level). However, the *NIRTC*(4, 4, 2) did not raise more revenue than the *RTC*(4, 4, 2). Our data also suggests that withholding information *and* restricting quantity is no better than conducting an RTC auction with only one of these features: the difference between the revenue of the *NIRTC*(4, 3, 2) and the revenues of the *NIRTC*(4, 4, 2) and the *RTC*(4, 3, 2) is *not* significant.

The expected revenue generated by an auction format is not the only criterion to judge its desirability. All other things being equal, most sellers would probably prefer auction formats in which the variability of revenue is low. Theoretically, both the restricted and the unrestricted right-to-choose auctions are expected to be less volatile than the standard good-by-good auction. For example, as we see in Table 3, at the equilibrium the variance of the revenues generated by the *RTC*(4, 4, 2) auction is 33.7 while it is 52.9 for the *GBG*(4, 2) auction. Although the right-to-choose auctions actually produced a higher variation of revenues than expected in theory, each generated a lower variance than both GBG auctions. A series of pair-wise F-tests reveal that the *RTC*(4, 3, 2) and *NIRTC*(4, 3, 2) are least volatile at the 1% level.[12] The difference between the variance of the *RTC*(4, 4, 2) and those of the two GBG auctions (the *GBG*(4, 2) and the *OGBG*(4, 2)) is not statistically significant at the 5% level. These findings provide further support that a seller faced with unknown demand may benefit from conducting RTC auctions with quantity restriction.

It is also important to note that the revenues in periods 9–16 are virtually identical to those raised in all 16 periods. For example, the revenues raised by the *RTC*(4, 4, 2) and the *RTC*(4, 3, 2) in periods 9–16 were 206.5 and 192.3 respectively, while those raised in all 16 periods were 203.7 and 188.7 respectively. Similarly, the revenue raised by the *OGBG*(4, 4, 2) in periods 9–16 is 179.1, whereas the average revenue over all 16 periods is 178.8. When comparing revenues in the last eight periods across auction designs, the difference between the *OGBG*(4, 2) revenue and that of the *RTC*(4, 4, 2) in periods 9–16 remains statistically significant at the 5% level. It is not the case that subjects behaved differently in the second part of the experiment.[13] Behavior did not converge to something closer to that predicted by standard theory.

We now turn to a comparison of the auctions in terms of efficiency. We focus on two measures of efficiency. First, we investigate the allocative efficiency of these auctions by looking at whether

[12] An exception is the comparison between *NIRTC*(4, 3, 2) and *NIRTC*(4, 4, 2) that is insignificant at $p = 0.11$.

[13] The revenues raised by the *NIRTC*(4, 4, 2) and *NIRTC*(4, 3, 2) auctions in periods 9–16, 197.5 and 178.3 respectively, are in the same ballpark as the revenues for all 16 periods (196.7 and 187.1).

the *available* goods are allocated to the bidders who value them the most. To measure allocative efficiency we simply count the fraction of times during the auction that the goods were sold to the highest valuation bidders. We call this measure *Ordinal Efficiency* since it only takes into account whether an optimal trade was made or not but not the value of the trade.

One drawback of ordinal efficiency is that it disregards the magnitude of welfare losses. In particular, this measure of efficiency does not distinguish between a good that is not sold to any buyer and a good that is not sold to the highest valuation buyer. We therefore propose a second measure of efficiency, called *Cardinal Efficiency*, which is defined as the ratio of "realized surplus" to "maximal surplus." By "realized surplus," we mean the sum of values of the winning bidders, whereas "maximal surplus" refers to the sum of values of the highest valuation bidders for each good. While Ordinal Efficiency reports whether or not a welfare loss was incurred, Cardinal Efficiency reports the magnitude of this loss. Table 4 presents our efficiency results.

Several features of this table are noteworthy. First, notice that the $RTC(4,4,2)$ and the $GBG(4,2)$ auctions are close to being fully efficient in the cardinal sense. Each practically achieves 100% efficiency. Second, note that they are less efficient in ordinal terms. This clearly indicates that if an inefficient allocation occurred it was more likely to have occurred when the subjects valuing the same good had similar values. This follows from the fact that inefficient allocations have equal weight under ordinal efficiency, whereas an inefficient allocation among subjects with close values has small effect under cardinal efficiency (i.e. it matters little for cardinal efficiency if a good is allocated to a subject who values it at 76 while his pair member values it at 77).

The NIRTC auctions, as expected, do rather poorly. For example, the ordinal efficiency of the $NIRTC(4,4,2)$ is only 66.8%, and the ordinal efficiency of the $NIRTC(4,3,2)$ is 56.3%, the lowest among all auction formats. The realized cardinal efficiencies of the $NIRTC(4,4,2)$ and $NIRTC(4,3,2)$ are somewhat higher: 79.9% and 71.5% respectively. This fact, coupled with the revenue results discussed above, suggests that NIRTC auctions are poor institutional choices for sellers in our type of environment.

We used Mann–Whitney tests to determine whether or not differences in efficiency scores are statistically significant. Under both notions of efficiency, the difference between the score of the $RTC(4,4,2)$ and those of $RTC(4,3,2)$ and $OGBG(4,2)$ is statistically significant at the 1% level. As expected, the two efficiency scores of the $RTC(4,4,2)$ are not statistically different from those of the $GBG(4,2)$, while those of the latter are significantly higher than the ordinal and cardinal scores of the $OGBG(4,2)$ (at the 1% level).[14]

Table 4
Efficiency

	Ordinal efficiency (%)		Cardinal efficiency (%)	
	Predicted	Actual	Predicted	Actual
$RTC(4,4,2)$	100.0	92.6	100.0	98.2
$RTC(4,3,2)$	75.0	69.7	84.9	83.4
$NIRTC(4,4,2)$	77.1	66.8	89.3	79.9
$NIRTC(4,3,2)$	65.1	56.3	78.2	71.5
$GBG(4,2)$	100.0	92.4	100.0	98.3
$OGBG(4,2)$	78.9	73.8	87.7	87.9

[14] As expected, the efficiency scores (ordinal and cardinal) of the $NIRTC(4,3,2)$ are significantly lower than those of any other auction format. A similar observation holds for the $NIRTC(4,4,2)$ with two exceptions: it is more efficient

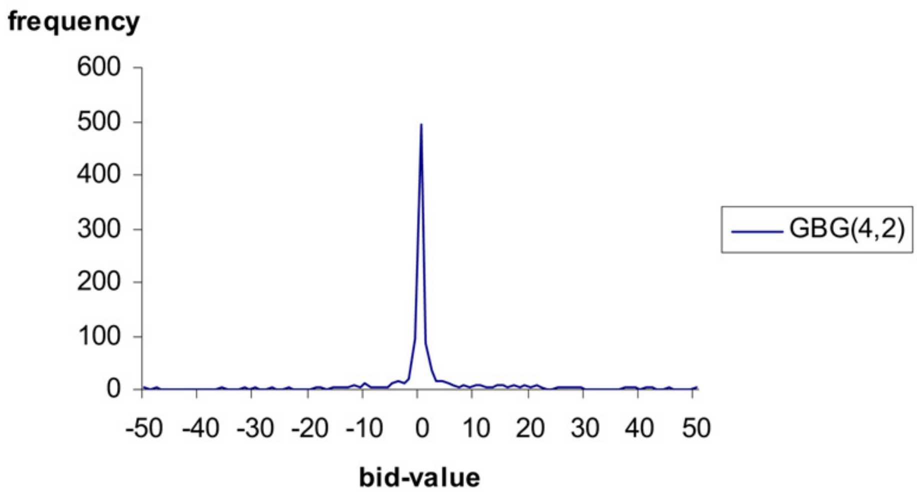
Table 4 highlights an interesting experimental finding. A recurring theme in the mechanism-design literature is the trade-off between efficiency and optimality. In contrast, we find that the highest revenue is in fact generated by an efficient auction: the unrestricted RTC auction.

5.2. *Bidding behavior*

In this section we offer an explanation for why our experimental RTC and NIRTC auctions outperformed their GBG and OGBG counterparts. First, we demonstrate that the superior performance of our RTC and NIRTC auctions was not due to a sub-optimal performance on the part of our GBG or OGBG auctions: the subjects in these auctions basically bid their value in accordance with the theory. Second, we argue that risk aversion *alone* cannot explain the high bids in *both* the RTC and NIRTC auctions. Finally, we propose a behavioral explanation for why the right-to-choose format (with/without information and with/without quantity restriction) induces the competitive bidding behavior observed in our experiment. We argue that consistent with the conventional wisdom of auctioneers, bidders are led to believe that the competition they face is fiercer than it actually is.

5.2.1. *Bidding behavior in GBG and OGBG auctions*

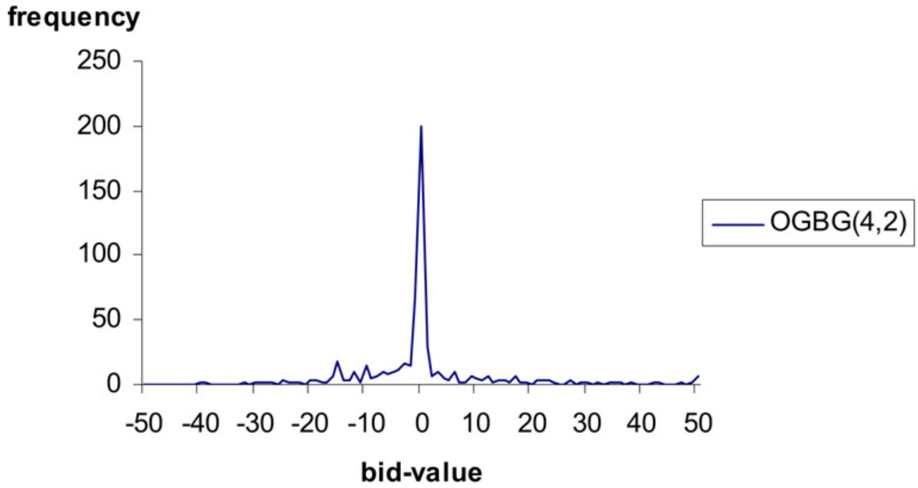
As stated above, it is important to note that our RTC auctions outperform the GBG auctions (with and without optimal reserve prices) despite the fact that, at least in those auctions, subjects bid according to the theory. Hence, the relatively good performance of our RTC auctions was not the result of sub-optimal behavior by subjects in our $GBG(4,2)$ or $OGBG(4,2)$ experiments. To illustrate this point, remember that in these auctions, given their second price nature, subjects are expected to bid their value. From the bid data we see that this is true in the sense that the median difference between bid and value in both of these auctions is zero while the mean difference between bid and value in the $GBG(4,2)$ and $OGBG(4,2)$ auctions are 1.21 and 0.40, respectively. Figures 1(a) and 1(b) present histograms where the variable on the x axis is the difference between the value a bidder received and his or her bid in the GBG and OGBG auctions



(a)

Fig. 1.

(ordinally and cardinally) than the $NIRTC(4,3,2)$ and its ordinal efficiency score is not statistically different from that of the $RTC(4,3,2)$.



(b)

Fig. 1.

respectively. Note that a clear majority of the bidders basically bid their values in the two auction formats. There are only very few bids that deviate substantially from value. Such behavior indicates that bidders in these auctions understand that it is weakly dominant to bid one's value.

5.2.2. Risk aversion in RTC and NIRTC auctions

The previous subsection presented clear evidence that the mean revenues in our experimental RTC and NIRTC auctions are substantially higher than the expected revenues in the SPBE of these auctions. This means that subjects in our experimental RTC and NIRTC auctions tend to submit higher bids than those predicted by our theoretical analysis. We first deal with the possibility that risk aversion explains the deviations of actual bids from the theoretical analysis that assumed bidders are risk-neutral. In our original RTC auctions (with and without quantity restriction), a risk averse bidder would bid higher than his risk neutral cohort (assuming both behave according to equilibrium). To see this, note that as long as one does not bid above his value in a RTC auction, a bidder can avoid incurring a loss. Hence, if a bidder decides to bid above the prescribed risk neutral equilibrium bid, he or she is simply trading off an increased probability of winning a good against the profit to be made conditional on winning. As is true of auctions in general, risk averse subjects deal with this trade-off by raising their bids. Hence, we would expect that risk averse subjects in RTC auctions would do exactly as we have observed them doing and bid above the predictions of the risk neutral equilibrium bid function. This suggests that risk aversion may potentially account for the high revenues generated by our RTC auctions.

In contrast, risk-averse bidders would bid *below* their risk-neutral bids in a NIRTC auction. To see why, note that a NIRTC auction is in essence a second-price auction in which the highest bidder of each phase wins a lottery that awards that bidder his most preferred good with some probability and nothing with the complementary probability. Consider then a bidder in the final phase of such an auction where there is, say, 50% chance of winning one's good and 50% chance of winning nothing. A risk-neutral bidder would bid half his value, but a risk-averse bidder would bid his certainty equivalent of the lottery, which is *strictly less* than half his value. As a result, we would expect the revenues in NIRTC auctions with risk-averse bidders to be strictly lower than the revenues generated by risk-neutral agents. However, this is not what we observe in the data.

To illustrate this point, suppose all bidders have the same CARA utility function of the form $U(x) = \frac{1-e^{-rx}}{r}$. With more than two phases, we were not able to derive analytical solutions for

Table 5
Theoretical and actual revenues

	$RTC(4,4,2)$	$RTC(4,3,2)$	$NIRTC(4,4,2)$	$NIRTC(4,3,2)$
Actual	203.7	188.7	196.7	187.1
CARA, $r = 0.07$	162.1	164.0	122.9	117.4
Risk neutral, $n = 2$	133.3	152.1	152.1	147.6
Risk neutral, as if $n = 6$	185.5	176.5	164.2	154.1

the equilibrium bids. Therefore, we computed the bids numerically for the parameter $r = 0.07$ that provided a good fit for the data of a previous auction experiment.[15] Figures 2(a)–2(d) show the predicted bids together with the average actual bids. For the RTC auctions, the bids predicted by the CARA model come reasonably close to the actual bids, although predicted bids are still below actual bids. The fit of the CARA predictions for the NIRTC auctions is much worse, though. There, predicted bids fall far below the actual bids, especially in the later phases of the auction.

Table 5 presents the revenues in the RTC and NIRTC auctions that can be expected for a CARA model with $r = 0.07$ in combination with the observed revenues and the revenues predicted by the risk neutral model (in the row labeled $n = 2$, the other row will be discussed later). As evident from the table, the revenues generated in the lab by the NIRTC auction are substantially above the revenues expected from risk-averse bidders. This exercise exemplifies the general problem for the risk aversion approach. To improve the fit of the RTC data, one needs to introduce risk aversion, but risk aversion decreases the fit of the data in the NIRTC auctions.

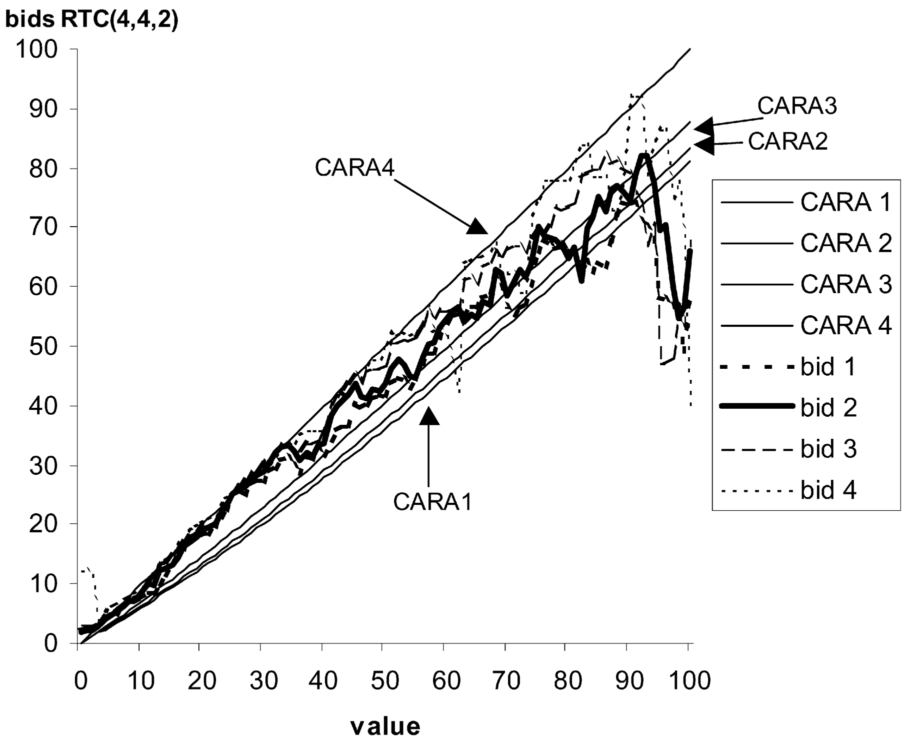
5.2.3. A possible explanation

Auctioneers praise the RTC procedure because it artificially creates competition between bidders who would otherwise refrain from competing. Subjects' comments in the informal debriefing after the experiment supported this auctioneers' intuition. In the RTC auctions as well as the NIRTC auctions, subjects mentioned that they thought they had participated in "extremely competitive games."

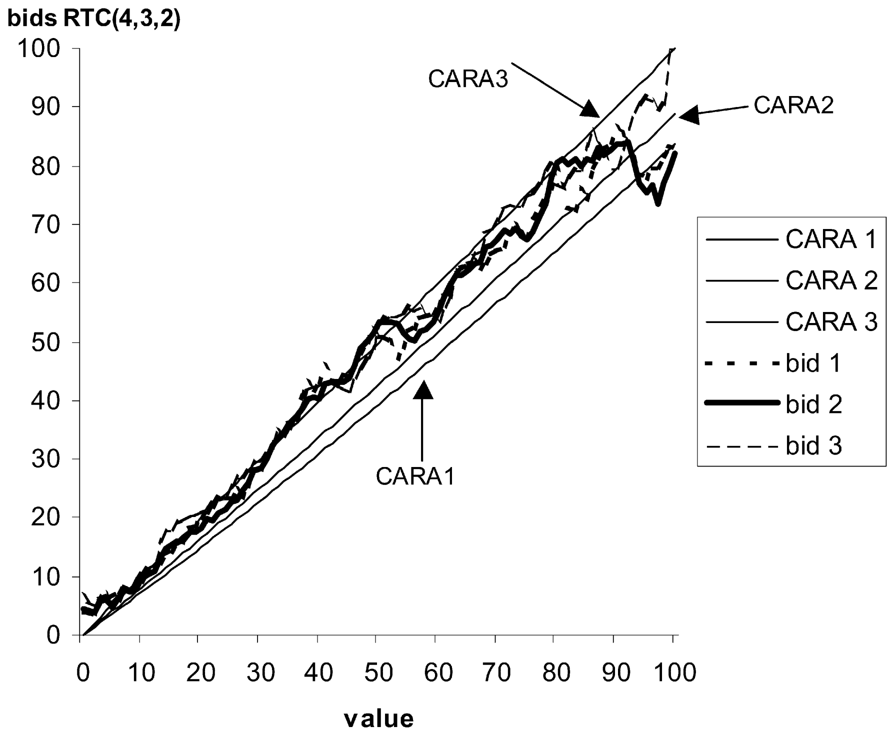
In theory, RTC and GBG auctions are revenue equivalent because despite the fact that there are many people in the auction, each bidder really only faces competition from one other person—the one who values his same good. This is the key to bidding behavior in theory. So the perception that these auctions increase competition is a false perception.

When subjects actually engage in these auctions, however, the key feature of the theory is obscured because they learn that unless they have received a very high value for their good, there is a significant chance that they would lose to their competitor. Such bidders come to the realization that in order to increase their chances of winning, they must try to snatch the good in an early phase of the auction. But in these phases bidders face competition by all other bidders since they must outbid all of them in order to win. This leads them to bid close to their value in early phases under the realization that even a small profit won early is better than none realized late. From their perspective, bidders with middling values face competition from more than just one bidder in the first phase, and tend to bid "as if" the number of buyers after their good is greater than one.

[15] Goeree and Offerman (2003) use the same utility function to analyze second-price private value auctions with value uncertainty. They report maximum likelihood estimates for r in the range of $[0.06, 0.08]$.



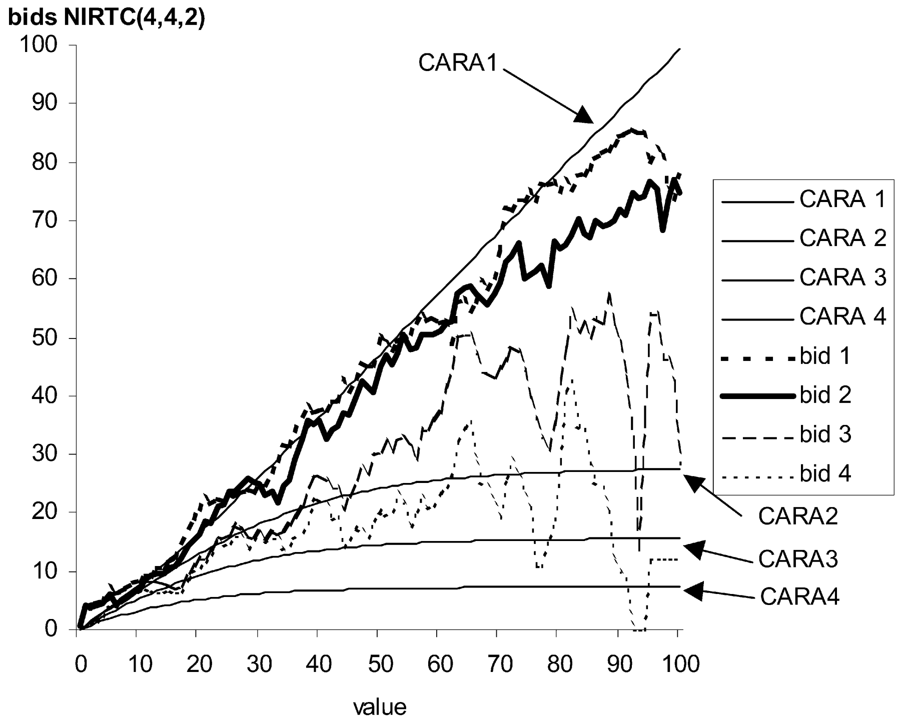
(a)



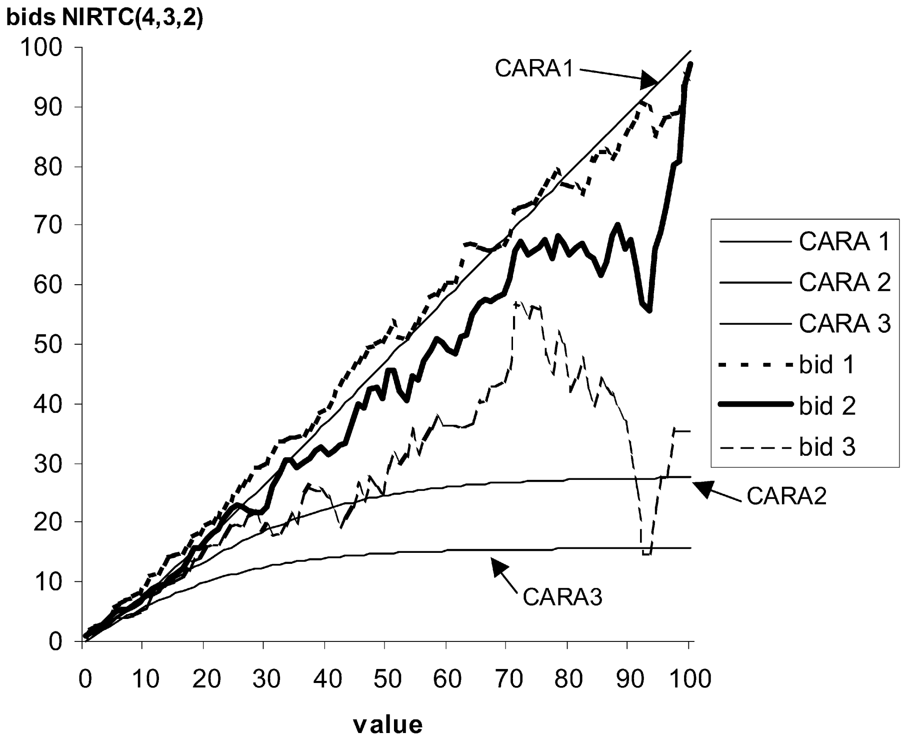
(b)

Notes. For every value the average of actual bids in the interval [value − 2, value + 2] is shown. Bid i refers to the bids in phase i. CARA j refers to the CARA predictions for phase j ($r = 0.07$).

Fig. 2.



(c)



(d)

Fig. 2. (*Continued*)

Bidders with very high values, on the other hand, tend to be confident that there is at least one other bidder with a value higher than their competitor. In addition, these bidders understand that by winning in later phases, they can obtain their good at a lower price. Hence, bidders with high valuations tend to underbid in the early phases—sometimes even below their lower-valued competitor—only to bid more aggressively in later phases in the hope of winning the good at a cheaper price. These high-value bidders are by far a minority, however, so the point is that for the vast majority of bidders they behave as if there were many bidders that must be beaten if they are to be successful in attaining a positive surplus.

A similar logic applies to the NIRTC auctions except here the motivation is slightly different. Here, especially in the first round, bidders realize that unless they have a very high value they are likely either not to win a good at all (as was true in the RTC case) or, even worse, win one that they do not want and therefore suffer a loss. This creates even more pressure on them to bid high in the first or second round where they have to beat many bidders to succeed.

A parsimonious way to model the auctioneers' wisdom and our subjects' experience is to assume that bidders bid *as if* there are more buyers who are after their good. That is, since there are eight bidders in total, each subject in an RTC auction behaves as if he compete with n other subjects over his good, where $2 < n \leqslant 8$.

To investigate this possibility, we perform the following exercise. For each of the four RTC formats we compute the equilibrium bids across the phases when $K = 4$ and $n \geqslant 2$. For each value of $n \geqslant 2$ we compute the mean squared error (MSE) between all bids observed in the experiment and the corresponding equilibrium bids. We then search for the value of n that minimizes the MSE. Table 6 presents the mean squared errors for $n = 2, \dots, 8$.[16]

Table 6 may be interpreted as saying that subjects in the $RTC(4, 4, 2)$ bid as if they coordinated on the SPBE of a $RTC(4, 4, 6)$ in the sense that $n = 6$ minimized the MSE for this auction format. Moreover, the MSE of $n = 6$ is lower than the MSE of $n = 2$ by more than 30%. Notice that if the exercise is carried out for each treatment separately, the mean squared error is consistently minimized for either $n = 6$ or $n = 7$. The absence of variance in the optimal estimate of n across treatments lends credibility to the idea that bidders bid as if there is a larger number of bidders interested in their good.[17]

To provide further evidence in support of our hypothesis that subjects bid as if they were facing fiercer competition than they actually did, we plotted the average bid in each phase for each auction format. We then compared these bids with the SPBE bid function of each phase computed

Table 6
Mean squared error bids: $(\text{bid} - \text{prediction})^2$

As if n	All	$RTC(4,4,2)$	$RTC(4,3,2)$	$NIRTC(4,4,2)$	$NIRTC(4,3,2)$
2	301.90	415.52	186.09	292.43	304.18
3	223.32	252.43	120.31	240.15	264.07
4	207.27	219.38	104.99	230.18	256.40
5	203.34	212.21	100.54	227.48	254.55
6	202.79	212.17	99.36	226.76	254.26
7	203.33	214.30	99.35	226.70	254.46
8	204.21	217.03	99.77	226.88	254.82

Notes. The table lists the mean squared error between bids and predictions for different "as if group sizes." The column "all" pools all cases of all treatments, the other columns present the MSE per treatment.

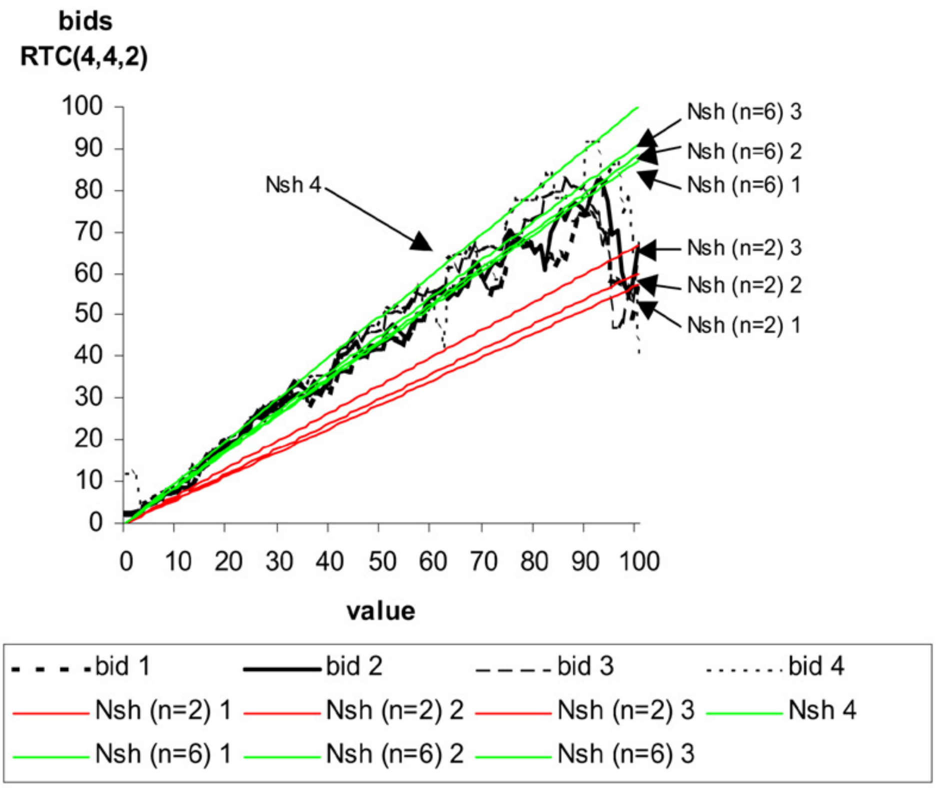
[16] As stated earlier, our approach here is equivalent to saying that subjects misperceive the probability that their good will be taken in each phase. However, subjects still believe that they all face the same competition. They do not realize that this belief, together with the above misperception, is not consistent with the total number of bidders in the auction.

[17] The assumption that bidders bid as if there is more competition than actually present does not alter the predictions for the (O)GBG auctions. There, bidders have a weakly dominating strategy to submit their values, independent of the perceived number of competitors.

once according to $n = 2$ and once according to the MSE-minimizing $n = 6$. The resulting graphs for each auction format are presented in Figs. 3(a)–3(d).

As evident from the graphs, the equilibrium bids computed for the MSE-minimizing n provide a good fit for the bidding patterns observed in the RTC auctions and the NIRTC auctions. Remarkably, the NIRTC auctions produce the opposite deviation from the $n = 2$ predictions. Here, first phase actual bids are *above* the $n = 2$ predictions while the last phase actual bids are *below* the $n = 2$ predictions (see Figs. 3(c) and 3(d)). Thus, the actual data fan-out compared to the $n = 2$ predictions. Again, this bias is consistent with the $n = 6$ predictions that also fan-out compared to the $n = 2$ predictions.[18]

Note that in Figs. 3(a)–3(d) the mean bid appears to be concave and non-monotonic. That is, for low and intermediate values actual bids increase approximately linearly in value as predicted by the model, but for high values the bids increase less than linearly and sometimes even decrease. This feature is consistent with our earlier explanation since high-valued bidders choose to wait in early rounds and take their chances winning the good later in the auction when its price

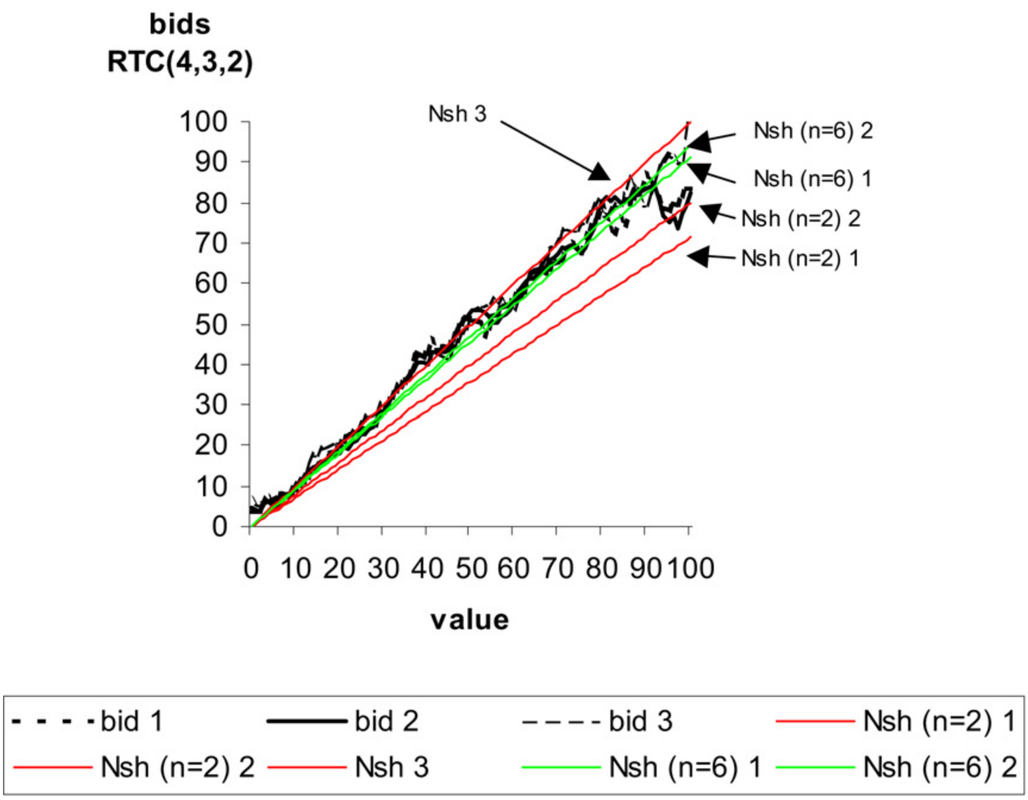


(a)

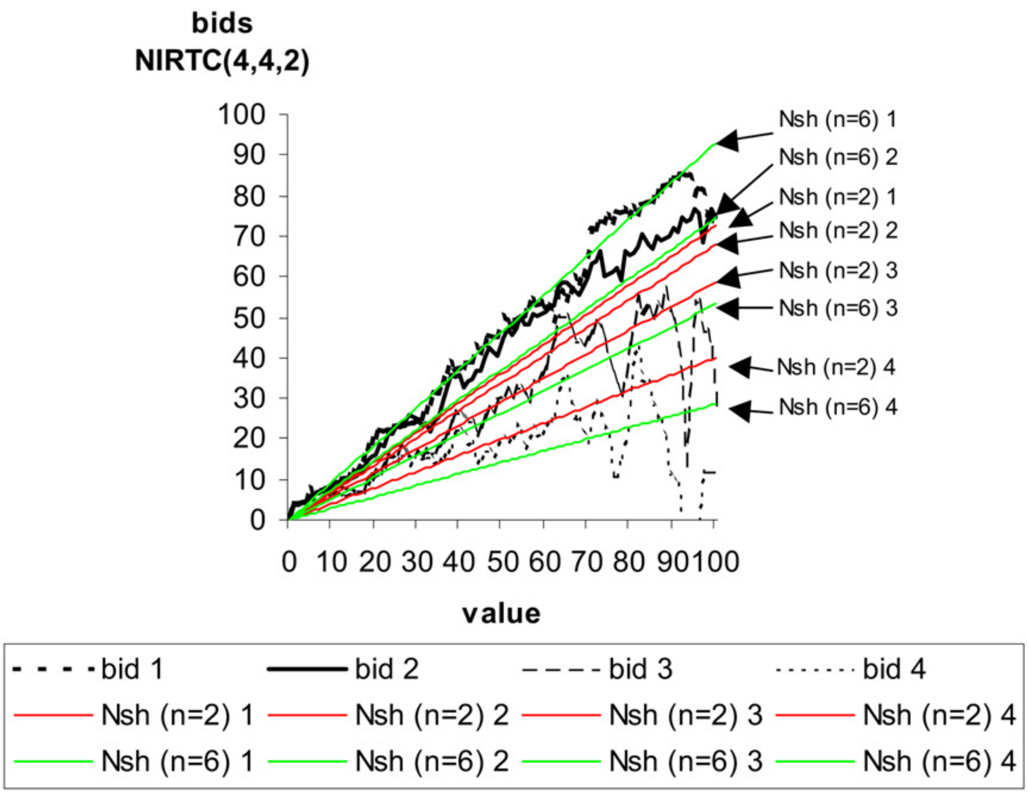
Notes. For every value the average of actual bids in the interval [value − 2, value + 2] is shown. Bid i refers to the bids in phase i. Nsh $(n = 2)$ j refers to the Nash prediction for phase j for the case $n = 2$. Nsh $(n = 6)$ k refers to the Nash prediction for phase k for the "as if case" $n = 6$. Nash predictions are identical in the final phase of the RTC auction.

Fig. 3.

[18] As mentioned in Section 2, an alternative explanation for the NIRTC bidding data is that subjects suffer from an attentional bias, as suggested by Salmon and Iachini (in press). According to this explanation, players underestimate the probability that they will receive a less preferred good. We do not explore this possibility here any further, because an attentional bias cannot explain the high bids observed in the RTC auctions, where remaining bidders know for sure that their preferred good is still available.

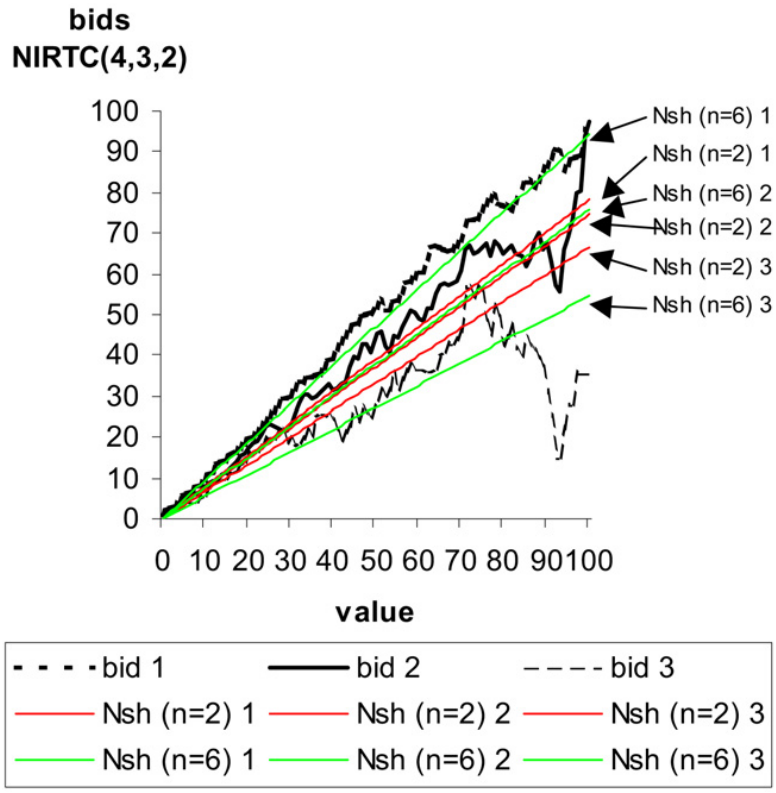


(b)



(c)

Fig. 3. (*Continued*)



(d)

Fig. 3. (*Continued*)

falls. The extreme concavity observed in the figure is a bit misleading and is caused by the fact that the mean bid is pulled down by a few bidders with extremely high values who deliberately tried to lose in the early phases by submitting extremely low bids.[19] Since the graphs present the mean bid at each value, these outliers, combined with the small numbers of people with such high values, have an exaggerated influence on the graphs.

Assuming bidders behave as if the number of buyers per good is the value of n that minimizes the MSE, what are the expected revenues in equilibrium? In Table 5 a row is added that lists the expected revenues of the treatments when $n = 6$. To be precise, in this row revenues are calculated for $RTC(4, 4, 6)$, $RTC(4, 3, 6)$, $NIRTC(4, 4, 6)$ and $NIRTC(4, 3, 6)$, respectively. Notice that of all three models considered in Table 5, the model that makes the as if $n = 6$ assumption comes closest to the actual revenue in each of the four treatments. Another important feature of the as if $n = 6$ model is that it gets the revenue comparison between the two RTC treatments right. In accordance with the data, for $n = 6$ the model predicts that quantity restriction leads to more aggressive bidding, but that the rise in bids is not enough to offset the loss of revenue from the last phase. In fact, for n higher than 2 quantity restriction starts raising revenue only when a higher number than 4 goods is offered for sale (see also Proposition 4). Our results underline the revenue enhancing potential of quantity restriction in RTC auctions, but they also show that quantity restriction works "less quickly" than predicted by the standard model.

[19] One extreme example of this is a bidder with a value of 96 who bid zero in the first and second phase, but bid his value in the third phase and won.

6. Concluding remarks

This paper examines a sequential mechanism for selling multiple goods, where in each phase the seller auctions a right-to-choose one of the available goods. We provide experimental evidence that the RTC auction induces an aggressive bidding behavior that generates a substantially higher revenue than that predicted by the theory. Moreover, in an experimental setting where the theoretical optimal auction is known, the RTC auction raised on average more revenue than the theoretical optimal auction.

Our findings have the following implications. First, sellers with multiple goods, each having a thin market, may benefit from thickening those markets via an RTC auction. In particular, in an environment where the optimal auction is difficult to compute (either because there is not enough information or because of multi-dimensionality), the RTC auction seems especially desirable since it is essentially "detail free": it can be executed independently of the distribution of buyers' valuations and regardless of whether or not the goods are substitutes for some of the buyers. Second, our findings may explain the popularity of the RTC format in environments where it is not the theoretical optimal auction. Our experimental results suggest that perhaps the RTC format was ultimately adopted because it proved to be better in terms of raising revenue than alternative procedures.

Acknowledgments

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Appendix A. Proofs

For ease of exposition, in what follows bidders will be labeled by their values, e.g., a bidder with value v will be referred to as "bidder v." We shall also use the notation $Y_i^{(m)}$ to denote the ith order statistic of m independent draws from [0, 1] (e.g., $Y_1^{(N_k)}$ denotes the highest order statistic among the remaining bidders in phase k).

Proof of Proposition 1. We need to show that the $RTC(K, K, n)$ auction has a SPBE of the following form:

$$b_k(v) = \frac{N_k - (K - k + 1)}{N_k - 1} v \tag{A.1}$$

where N_k denotes the number of bidders in round k.

Clearly, the final phase of the auction, phase K, is simply a standard second-price auction. Hence, $b_K(v)$ should be equal to v. Indeed, by letting $k = K$ in Eq. (A.1) we obtain

$$b_K(v) = \frac{N_k - 1}{N_k - 1} v = v.$$

It remains to show that (A.1) holds for all $k < K$ a bidder with valuation v. Suppose that in all stages after stage k, all bidders use the equilibrium strategies, i.e., they bid $\beta_j v$ where $\beta_j = [N_j - (K - j + 1)]/(N_j - 1)$. Suppose a bidder with valuation v bids b in stage k. Assuming all other bidders use their equilibrium strategies, this bidder wins if, and only if, the highest valuation among the remaining bidders is at most b/β_k. Hence, the probability of winning at stage k for this bidder is $(b/\beta_k)^{N_k-1}$. We may therefore compute the expected payoff of this bidder as follows:

$$\left(\frac{b}{\beta_k}\right)^{N_k-1}\left[v - \beta_k E\left(Y_1^{N_k-1} \mid Y_1^{N_k-1} \leqslant \frac{b}{\beta_k}\right)\right] \\ + \left[1 - \left(\frac{b}{\beta_k}\right)^{N_k-1}\right]\left[\frac{n(K-k)}{n(K-k+1)-1}\right]\left[v - \beta_{k+1} E\left(Y_1^{N_{k+1}-1} \mid Y_1^{N_{k+1}-1} \leqslant v\right)\right].$$

Writing the conditional expectations and taking the first order condition with respect to b gives:

$$0 = (N_k - 1)\left(\frac{b}{\beta_k}\right)^{N_k-2}\left(\frac{1}{\beta_k}\right)\left[v - b\left(\frac{N_k - 1}{N_k}\right)\right] - \left(\frac{N_k - 1}{N_k}\right)\left(\frac{b}{\beta_k}\right)^{N_k-1} \\ - (N_k - 1)\left[\frac{n(K-k)}{n(K-k+1)-1}\right]\left(\frac{b}{\beta_k}\right)^{N_k-2}\left(\frac{1}{\beta_k}\right)\left[v - \beta_{k+1} v\left(\frac{N_{k+1} - 1}{N_{k+1}}\right)\right].$$

Simplifying this expression, we obtain:

$$b = v\left[1 - \left(\frac{N_{k+1}}{N_k - 1}\right)\left(1 - \beta_{k+1} v\left(\frac{N_{k+1} - 1}{N_{k+1}}\right)\right)\right].$$

Equilibrium requires $b = \beta_k v$. Hence, the above difference equation can be solved, yielding (A.1). □

Proof of Proposition 2. When bidders bid their value in the $GBG(K, n)$, and when they follow the SPBE of the $RTC(K, K, n)$, as given in Proposition 1, then (1) the allocation of goods in both auctions is efficient, and (2) the expected payment of a bidder with value zero is zero. Hence, by the Revenue-Equivalence-Theorem, both auctions yield the same expected revenue. □

Proof of Proposition 3. The proof is essentially the same as the proof of Proposition 1. The only change is that $b_{K-1}(v) = v$, given that round $K - 1$ is the final round. □

Proof of Proposition 4. We begin by introducing a few helpful notations:

- $\mathcal{P}$ denotes the set of all partitions of nK order statistics into K groups of n. An element in $\mathcal{P}$ (i.e., a particular partition) is denoted P. If the ith order statistic is in the same group as the jth order statistic, then we say that the ith and jth order statistics demand the same good.
- Let $\beta_k^K v$ be the equilibrium bid function for $RTC(K, K, n)$.
- Let $\beta_k^{K-1} v$ be the equilibrium bid function for $RTC(K, K - 1, n)$.

Fix some partition $P \in \mathcal{P}$. Suppose we run both an $RTC(K, K, n)$ and an $RTC(K, K - 1, n)$ for this given P. Because the partition is held fixed, the set of order statistics who win is exactly the same in $RTC(K, K, n)$ and in $RTC(K, K - 1, n)$. Moreover, the order statistic, whose bid is paid in each of the first $K - 1$ rounds, is also the same in both auctions. Let $B^K(P)$ be the set of expected prices paid in the first $K-1$ rounds of $RTC(K,K,n)$. Similarly, let $B^{K-1}(P)$ be the set of expected prices paid in the first $K-1$ rounds of $RTC(K,K-1,n)$. Then

$$B^K(P)=\left\{\beta_1^K\left(\frac{N-\theta_1^P}{N+1}\right),\dots,\beta_{K-1}^K\left(\frac{N-\theta_{K-1}^P}{N+1}\right)\right\},$$
$$B^{K-1}(P)=\left\{\beta_1^{K-1}\left(\frac{N-\theta_1^P}{N+1}\right),\dots,\beta_{K-1}^{K-1}\left(\frac{N-\theta_{K-1}^P}{N+1}\right)\right\}$$

where $\frac{N-\theta_k^P}{N+1}$ is the θ_k^P+1 order statistic, such that $\theta_1^P=1$ and $n\geqslant\theta_{k+1}^P-\theta_k^P\geqslant 1$. We define $\Theta(P)\equiv(\theta_1^P,\dots,\theta_{K-1}^P)$.

For the given partition P, the total expected gain from quantity restriction is equal to the following sum:

$$\sum_{k=1}^{K-1}\left(\beta_k^{K-1}-\beta_k^K\right)\left(\frac{N-\theta_k^P}{N+1}\right). \tag{A.2}$$

While the expected loss is equal to $\frac{N-\theta_K}{N+1}$. From Propositions 2.1 and 2.2 it follows that $\beta_k^{K-1}-\beta_k^K=\frac{1}{N_k-1}$. Hence, expression (A.2) can be rewritten as follows:

$$\sum_{k=1}^{K-1}\left(\frac{1}{N_k-1}\right)\left(\frac{N-\theta_k^P}{N+1}\right). \tag{A.3}$$

Given P, quantity restriction *raises* expected revenue if and only if

$$\sum_{k=1}^{K-1}\left(\frac{1}{N_k-1}\right)\left(\frac{N-\theta_k^P}{N+1}\right)>\frac{N-\theta_K^P}{N+1}. \tag{A.4}$$

Multiplying both sides of (A.4) by $N+1$, we obtain that inequality (A.4) holds if and only if

$$\sum_{k=1}^{K-1}\left(\frac{N-\theta_k^P}{N_k-1}\right)>N-\theta_K^P. \tag{A.5}$$

We denote the LHS and RHS of (A.5) by $G(P)$ and $L(P)$ respectively. Hence, given P, the expected gain from quantity restriction is at least as large as the expected loss if and only if $G(P)\geqslant L(P)$.

Let $\mathcal{P}^j\subset\mathcal{P}$ be the set of partitions with the property that $\theta_K^P=K+j$ for every $P\in\mathcal{P}^j$, where $j\in\{0,1,\dots,(K-1)n+1\}$. Note that for any j every $P\in\mathcal{P}^j$ satisfies $\theta_k^P\leqslant k+j$ for $1\leqslant k\leqslant K-1$. This implies that for any j and any $P\in\mathcal{P}^j$, we have

$$G(P)\geqslant\sum_{k=1}^{K-1}\left[\frac{N-(k+j)}{N_k-1}\right]=\sum_{i=1}^{K-1}\frac{nK-(K-i+j)}{(i+1)n-1},$$
$$L(P)=N-K-j=(n-1)K-j.$$

Hence, $G(P)\geqslant L(P)$ if and only if

$$\sum_{i=1}^{K-1}\frac{(n-1)K-j}{(i+1)n-1}+\sum_{i=1}^{K-1}\frac{i}{(i+1)n-1}\geqslant(n-1)K-j.$$

Let $\Psi(K,n) \equiv [\sum_{i=1}^{K-1} \frac{1}{(i+1)n-1}] - 1$. Then the above inequality holds if and only if

$$\big[(n-1)K - j\big]\Psi(K,n) + \sum_{i=1}^{K-1} \frac{i}{(i+1)n-1} \geqslant 0. \tag{A.6}$$

Define $K^*(n)$ to be the smallest positive integer that satisfies $\Psi(K,n) \geqslant 1$. Such an integer exists because $\Psi(K,n)$ increases with K and tends to infinity as $K \to \infty$. It follows that inequality (A.6) holds for all $K \geqslant K^*(n)$. Note that because $\Psi(K,n)$ decreases with n, the integer $K^*(n)$ increases with n.

We now turn to show that when $n = 2$ (as is the case in our experimental design) quantity restriction raises expected revenue for all $K > 2$. First, it is straightforward to verify that for $K = 3$, selling two out of three goods raises more expected revenue than selling all three goods ($R(3,2,2) = \frac{38}{35}$, while $R(3,3,2) = 1$). It remains to show that quantity restriction raises expected revenue for all $K > 3$. We begin by identifying the set of partitions for which quantity restriction lowers expected revenue.

Lemma A.1. $G(P) < L(P)$ *only if* $P \in \mathcal{P}^0$.

Proof. Assume not. Then there exists a partition $P \in \mathcal{P} \backslash \mathcal{P}^0$ for which $G(P) < L(P)$. First, we claim that for every $P \in \mathcal{P}$ and $1 \leqslant k \leqslant K-1$ the following must hold: $\theta_k^P \leqslant 2k-1$. This follows from the fact that $\theta_1^P = 1$ and $n \geqslant \theta_{k+1}^P - \theta_k^P \geqslant 1$ for every P and k. Next, we claim that for all $P \in \mathcal{P}$ and $1 \leqslant k \leqslant K-1$ we have $\frac{N-\theta_k^P}{N_k-1} \geqslant 1$. Suppose not. Then $2K - \theta_k^P < 2(K-k+1) - 1$, which implies that $\theta_k^P > 2k-1$, a contradiction. Therefore, $G(P) \geqslant K-1$ for all $P \in \mathcal{P}$, where equality holds if $K = 2$. Because each $P \in \mathcal{P} \backslash \mathcal{P}^0$ satisfies $L(P) \leqslant K-1$, we have that $G(P) \geqslant L(P)$, in contradiction to our initial assumption. □

By Lemma A.1, there are exactly $(K-1)!$ partitions for which $G(P) < L(P)$. Moreover, for each of these partitions, $L(P) - G(P) = K - G^0$. Because each partition P is equally likely, $E_{P\in\mathcal{P}}[G(P) - L(P)] > 0$ if there exist at least $(K-1)!$ partitions for which $G(P) - L(P) \geqslant K - G^0$, where the inequality is strict for at least one of these partitions.

Let $\mathcal{P}^* \subset \mathcal{P}^K$ be a set of partitions with the property that for every $P \in \mathcal{P}^*$, $\Theta(P) = (1, \ldots, K-1)$. This set contains $(K-1)!$ partitions. In addition, for every $P \in \mathcal{P}^*$, $G(P) - L(P) = G^0 - 1$. Thus, $G^0 - 1 > K - G^0$ if and only if $G^0 > \frac{1}{2}(K+1)$. If $K > 3$, then the latter inequality holds because $G^0 \geqslant K-1$. □

Proof of Proposition 5. We need to show that for all $q < K$, the *NIRTC*$(K, K-q, n)$ auction has a SPBE in which bidders use a linear bid function in every round, where the bidding coefficients $(\beta_k)_{k=1}^{K-q}$ are given by

$$\beta_{K-q} = \frac{n(q+1)}{(n-1)K + q + 1} \tag{A.7}$$

and the unique solution to the following system of difference equations

$$\frac{N(n-1)}{N_k N_{k+1}} = \beta_k - \frac{N_{k+1}-1}{N_{k+1}} \beta_{k+1}. \tag{A.8}$$

The proof proceeds in three steps. First, we derive p_k, the probability that a bidder's good is still available in round k. Second, we use the expression for p_{K-q} to derive the bid in the last round. Third, we verify that the bid function (A.8) balances the gains and losses from bidding just above or just below the equilibrium bid, assuming all other bidders use (A.8).

STEP 1. Consider some bidder i in round k of the auction. There were $k-1$ winners in previous rounds. For each one of the $k-1$ winners, there are $n-1$ bidders among the remaining $nK-k+1$ bidders whose good was taken away.[20] There are $\binom{nK-k+1}{(n-1)(k-1)}$ possible combinations of selecting from among the $nK-k+1$ bidders in round k, the $n-1$ bidders whose good was taken away. Of these combinations, $\binom{nK-k}{(n-1)(k-1)}$ do not include bidder i. Therefore, the probability that bidder i's good is still present in round k is

$$p_k = \frac{\binom{nK-k}{(n-1)(k-1)}}{\binom{nK-k+1}{(n-1)(k-1)}} = \frac{n(K-k+1)}{nK-k+1}. \tag{A.9}$$

STEP 2. The subgame that begins with the final round of the auction (round n) is a one-shot second price auction. Therefore, it is a weakly dominating strategy for each bidder to bid his expected value, i.e., his value v multiplied by the probability that his good is still available. Using (A.9), we have

$$b_{K-q}(v) = p_{K-q}v = \frac{n(q+1)}{(n-1)K+q+1}v. \tag{A.10}$$

STEP 3. Suppose that in each phase $k \in \{1,\dots,K-q\}$, bidders use a continuous increasing bid function $b_k(v)$ that maps $[0,1]$ onto $[0,1]$. Consider the subgame that begins in some phase k of the auction. The SPBE bid in this phase is the highest price a bidder is willing to pay, conditional on the "pivotal" event that both he and another bidder have the highest valuation. By the same argument given in the proof of Proposition 1, the willingness to pay of the highest valuation bidder in round k must therefore be equal to the difference between the expected value of winning in round k and the expected value of winning in the next round (where both expectations are computed conditional on the above pivotal event). Hence,

$$p_k v - b_k(v) = p_{k+1}v - b_{k+1}\left[E\left(Y_1^{(nK-k-1)} \mid Y_1^{(nK-k-1)} < v\right)\right]. \tag{A.11}$$

From Step 1 and our assumption that bidders' values are independently drawn from the uniform distribution on $[0,1]$, it follows that Eq. (A.11) can be rewritten as follows:

$$\left[\frac{n(K-k+1)}{nK-k+1}\right]v - b_k(v) = \left[\frac{n(K-k)}{nK-k}\right]v - b_{k+1}\left(\frac{nK-k-1}{nK-k}v\right). \tag{A.12}$$

Given Step 2, we can solve for $b_{K-q-1}(v)$ by substituting (A.10) for $b_{K-q}(v)$. Proceeding inductively, we can solve for $b_k(v)$ for $k = 1,\dots,K-q-1$. It is easy to see that for every k, the bidding function has the linear form $b_k(v) = \beta_k v$. Moreover, for every k the coefficient β_k is obtained by solving a linear equation, hence there is a unique solution for each β_k. By substituting $\beta_k v$ for $b_k(v)$ in (A.12) and rearranging we obtain Eq. (A.8). □

Proof of Proposition 6. By Proposition 3, it suffices to compare the expected revenue of $NIRTC(K,K,2)$ with that of $GBG(K,n)$. Let β_k^N denote the coefficient of the linear bidding strategies employed in the SPBE of $NIRTC(K,K,n)$.

[20] To see why, suppose $K=3$ and $n=2$. Consider the winner in the first round. Suppose this winner takes the good of the winner in the next round. Then the winner in round 2 will necessarily take the good of two remaining bidders. Thus, the winners in the first two rounds have taken away a good desired by two other bidders.

From Proposition 5 it follows that for a NIRTC auction with *no* quantity restriction, the bidding coefficients in each phase may be expressed by the following formula:

$$\beta_k^N = \frac{N(n-1)}{N_{k+1}} \sum_{i=k}^{K} \frac{1}{N_i}.$$

Therefore, the expected revenue of $NIRTC(K,K,n)$ is given by

$$\sum_{k=1}^{K} \beta_k^N \left(\frac{N-k}{N+1}\right) = \frac{N(n-1)}{N+1} \sum_{k=1}^{K} \frac{k}{N_k}. \tag{A.13}$$

It follows that the expected revenue of $NIRTC(K,K,n)$ is greater than the expected revenue of $GBG(K,n)$ if, and only if,

$$\frac{N(n-1)}{N+1} \sum_{k=1}^{K} \frac{k}{N_k} > \left(\frac{n-1}{n+1}\right) K. \tag{A.14}$$

For $n = 2$ inequality (A.14) becomes:

$$\frac{2K}{2K+1} \sum_{k=1}^{K} \frac{k}{2K-k+1} > \frac{K}{3}. \tag{A.15}$$

This inequality holds if, and only if,

$$\sum_{k=1}^{K} \frac{k}{2K-k+1} > \frac{K}{3} + \frac{1}{6}. \tag{A.16}$$

To show that this inequality (A.16) holds for all $K \geqslant 2$ rewrite the LHS of *this* inequality as follows:

$$\begin{aligned}\sum_{k=1}^{K} \frac{k}{2K-k+1} &= \sum_{k=1}^{K} \frac{(-2K+k-1)+(2K+1)}{2K-k+1} \\ &= -K + (2K+1) \sum_{k=1}^{K} \frac{1}{2K-k+1}.\end{aligned} \tag{A.17}$$

Using (A.17) we obtain that inequality (A.16) holds if, and only if,

$$\sum_{k=1}^{K} \frac{1}{2K-k+1} > \frac{8K+1}{6(2K+1)}.$$

The above inequality can be simplified further as follows:

$$\sum_{j=K+1}^{2K} \frac{1}{j} > \frac{2}{3} - \frac{1}{4K+2}. \tag{A.18}$$

We now proceeds by induction on K. It is easy to verify that (A.18) holds for $K = 2$. Assume it holds for some $K > 2$. We wish to show that this inequality also holds for $K + 1$. In order to show this it suffices to prove that when K is raised to $K + 1$, the net increase to the LHS is greater than the net increase to the RHS. Thus, to prove the inductive step we need to establish that

$$\frac{1}{2K+2}+\frac{1}{2K+1}-\frac{1}{K+1}>\frac{-1}{4K+6}+\frac{1}{4K+2}$$
$$\Updownarrow$$
$$\frac{1}{2(2K+1)}-\frac{1}{2(K+1)}>\frac{-1}{2(2K+3)}$$
$$\Updownarrow$$
$$\frac{1}{2}\Big(\frac{1}{2K+1}+\frac{1}{2K+3}\Big)>\frac{1}{2K+2}. \tag{A.19}$$

Because the function $\frac{1}{2K+i}$ is convex for $i=1,2,\ldots,$ it follows that inequality (A.19) must hold. $\square$

Proof of Proposition 7. Let $b_k^q(v)$ denote the symmetric equilibrium bidding function in phase k of $NIRTC(K,K-q,n)$. Because the highest $K-q$ bidders win in $NIRTC(K,K-q,n)$, we have that $R(K,K-q-1,n)-R(K,K-q,n)$ is equal to the following expression:

$$\sum_{k=1}^{K-q-1}\big(\beta_k^{q+1}-\beta_k^q\big)\left(\frac{N_k-1}{N+1}\right)-\beta_{K-q}^q\left(\frac{N-(K-q)}{N+1}\right) \tag{A.20}$$

where β_k^{q+1} and β_k^q are the bidding coefficients of the linear bidding functions $b_k^q(v)$ and $b_k^{q+1}(v)$. We now show that expression (A.20) is negative.

By (A.11), β_{K-q-1}^q and β_{K-q}^q must satisfy the following equation:

$$p_{K-q-1}v-\beta_{K-q-1}^q v=p_{K-q}v-\beta_{K-q}^q\left[\frac{N_{K-q}-1}{N_{K-q}}\right]v. \tag{A.21}$$

By (A.10) it follows that $p_{K-q-1}=\beta_{K-q-1}^{q+1}$ and $\beta_{K-q}^q=p_{K-q}$ Thus, Eq. (A.21) can be rewritten as follows:

$$\beta_{K-q-1}^{q+1}-\beta_{K-q-1}^q=\frac{\beta_{K-q}^q}{N_{K-q}}. \tag{A.22}$$

Hence,

$$\big(\beta_{K-q-1}^{q+1}-\beta_{K-q-1}^q\big)(N_{K-q}-1)=\left(\frac{N_{K-q}-1}{N_{K-q}}\right)\beta_{K-q}^q.$$

Using (A.11) we have that for $k<K-q-1$,

$$p_k v-\beta_k^q v=p_{k+1}v-\beta_{k+1}^q\left(\frac{N_{k+1}-1}{N_{k+1}}\right)v, \tag{A.23}$$

$$p_k v-\beta_k^{q+1} v=p_{k+1}v-\beta_{k+1}^{q+1}\left(\frac{N_{k+1}-1}{N_{k+1}}\right)v. \tag{A.24}$$

Thus, subtracting (A.24) from (A.23) we obtain

$$\beta_k^{q+1}-\beta_k^q=\big(\beta_{k+1}^{q+1}-\beta_{k+1}^q\big)\left(\frac{N_{k+1}-1}{N_{k+1}}\right). \tag{A.25}$$

Hence, using (A.22) we have that for all $k \leqslant K - q - 1$,

$$\beta_k^{q+1} - \beta_k^q = \left(\frac{\beta_{K-q}^q}{N_{K-q}}\right)\Pi_{l=k+1}^{K-q-1}\left(\frac{N_l - 1}{N_l}\right). \tag{A.26}$$

Note that both the $NIRTC(K, K-q, n)$ and the $NIRTC(K, K-q-1, n)$ share the following property for $k \leqslant K-q-1$: $N_k - 1 = N_{k+1}$. Using this property, Eq. (A.26) becomes:

$$\beta_k^{q+1} - \beta_k^q = \frac{\beta_{K-q}^q}{N_{k+1}}. \tag{A.27}$$

Multiplying both sides of Eq. (A.27) by $N_k - 1$ and summing from $k = 1$ to $k = K - q - 1$ we obtain

$$\begin{aligned}\sum_{k=1}^{K-q-1} \left(\beta_k^{q+1} - \beta_k^q\right)(N_k - 1) &= \sum_{k=1}^{K-q-1} \left(\frac{N_k - 1}{N_{k+1}}\right)\beta_{K-q}^q \\ &= (K-q-1)\beta_{K-q}^q \\ &< \left[(n-1)K + q\right]\beta_{K-q}^q.\end{aligned} \tag{A.28}$$

By dividing both sides of the last inequality in (A.28) by $N + 1$ we obtain (A.20). □

Proof of Proposition 8. Consider the market for good k. Our assumptions on the demand in this market satisfy the assumptions made in Riley and Samuelson (1981). A straightforward application of their model implies that the optimal mechanism in the market for good k can be implemented by a second price auction with a reserve price of $\frac{1}{2}$.

Now consider any general mechanism Γ for selling all the K goods. Because all bidders draw the value for their preferred good independently from the same distribution, and because the number of buyers who value each good is the same, the expected revenue obtained by Γ must be equal to nK times the expected payment made by a single bidder in that mechanism. This means that by an appropriate choice of a reserve price (which may be random) one can generate $\frac{1}{K}$ of the expected revenue obtained by Γ by using a second-price auction for good k. But this means, by the argument in the previous paragraph, that the highest expected revenue is obtained by conducting K separate second-price auctions with a reserve-price of $\frac{1}{2}$. □

Appendix B. Instructions *RTC*(4, 3, 2)

Welcome to this experiment on decision-making! You can make money in this experiment. Read the instructions carefully. There is paper and a pen on your table. You can use these during the experiment. Before the experiment starts, we will hand out a summary of the instructions and there will be one practice period.

THE EXPERIMENT

You will earn points in the experiment by purchasing a good you value in a market. At the end of the experiment your points will be exchanged to dollars. Each 15 points will yield 1 dollar. At the beginning of the experiment you will receive a starting capital of 150 points that you will not have to pay back at the end of the experiment. You will also be able to earn more money as the experiment progresses. The experiment consists of 16 periods. Your total earnings in the experiment will be equal to the sum of the starting capital and your earnings in all 16 periods.

Each period you will be allocated to the same group of eight persons and within each period there will be three phases. Your earnings will be determined by your own choices and the choices of the other participants in your group. In each group four fictitious goods will be available for sale in each period: good A, good B, good C and good D.

VALUES OF THE GOODS

Each participant will want to buy only one of the goods in a period: the value for this good to him or her will lie between 0 points and 100 points, and each number between 0 and 100 is equally likely. That is, the value of the good is equally likely to be 25 as it is to be 100 as it is to be 51 etc. The other goods have no value (= 0 points) to the participant. Each participant will receive a different value for her or his preferred good (that is, the one good for which she or he has a positive value). The value of the preferred good of the one participant does not depend on the values of the preferred goods of the other participants. The value of your preferred good is therefore (very) likely different from those of others. At the start of a period you will get to know which one is your preferred good and how much you value it. You will not know the values of the preferred goods of other participants, other participants will not know the value of your preferred good. Among the seven other participants in the group there will be one other who also values the same good as you. This means that there is exactly one other person in your group who wants to buy the same good as you do and his or her value is also determined randomly from the interval between 0 and 100.

Which good a participant prefers changes (randomly) from period to period. This implies that the person who prefers the same good as you do changes (randomly) from period to period. Each participant also receives a new value for the preferred good in each period. The value for a preferred good in the one period will not depend on the value for the preferred good in any other period.

SALE OF THE GOODS

Rather than sell the goods one by one, the market you participate in will, in each period, sell 'rights to choose' one by one. If in any period you win one of the rights to choose you will be able to choose which of the goods remaining at that time you want. To be more precise, each period consists of three phases. In each phase a 'right to choose' is sold to the highest bidder. In the first phase all eight bidders in a group will submit a bid for the first right to choose. The highest of these eight bidders wins the first right to choose and chooses the good that she or he prefers. At the end of the first phase, every bidder will be informed whether she or he won the first phase or not. The winner of the first phase and the person that prefers the same good as the winner will no longer participate in the remaining phases of this period and will have to wait until the next period starts.

Then the second phase starts, where the remaining six bidders (whose goods are still unsold) submit a new bid for the second right to choose. At the end of this phase, each bidder is informed whether he or she was the winner. The highest bidder wins and chooses the good that he or she likes. This winner and the one other buyer who wants the same good as the winner will no longer participate in the remaining phase of this period and will have to wait until the start of the next period. In the third and final phase the remaining four bidders submit a new bid for the third right to choose. The highest bidder wins the third right to choose and selects the good that he or she likes.

Notice that only three of the available four goods are sold in a period. Which goods are sold depends on the bids of the participants.

PRICES OF THE GOODS

In each phase, the winner of a good pays a price that depends on the bids of that phase. Each participant submits a 'drop out price': this amount reflects what the participant maximally wants to pay for the right to choose in that phase. This drop out price has to be an integer number between 0 and 100 points. The winner and winning price in any phase is determined as follows: First, the computer raises the price from 0 to 100 points. If the price reaches the 'drop out price' of a participant, this participant drops out and will not win the right to choose in the current phase. This process continues until the level where all but one participant have dropped out. The remaining bidder wins the right to choose and pays a price equal to this level. Notice that in this way the price will be equal to the second highest submitted drop out price.

If two (or more) participants have submitted the same drop out price which happens to be the highest, then one of these bidders will be randomly selected. Only in this case the winner pays a price equal to the own submitted drop out price.

The buyer of a right to choose will automatically receive the preferred good. The profit to the bidder from winning will be equal to her or his value minus the price she or he pays, so profit = (value-price). The only person who is told the price at which a good was sold in a particular phase is the winner of that phase. Participants that do not buy a right in a period receive a profit of 0 in that period.

Notice that the highest bidder in a phase can make a loss if she or he pays a price higher than her or his value for the good. This can only happen if the bidder submits a drop out price higher than her or his value, because in that case the second highest drop out price may also be higher than this value. For example, suppose Bob whose value for good A is 10 submits a drop out price of 15. If 15 is the highest drop out price and the second highest drop out price is 12, Bob wins goods A but incurs a loss of 2.

EXAMPLE

The procedure to sell the goods is now illustrated with an example. THE NUMBERS IN THE EXAMPLE ARE ARBITRARILY CHOSEN.

Assume that the winners of the first two phases have selected goods A and C. Then the four bidders who prefer either good B or good D bid in the third phase for the third and final right to choose. Assume that Bob submits a drop out price of 44, Arthur submits a drop out price of 23, Lisa submits a drop out price of 39, and Susan submits a drop out price of 59. Say that Bob and Arthur value the good B while Susan and Lisa value good D. Then the result will be as follows. The computer raises the price from 0. At a price of 23 Arthur drops, at a price of 39 Lisa drops and at a price of 44 Bob drops. The remaining bidder Susan wins her good and pays a price equal to 44. If Susan happened to value the good at 70, her profit would be $70 - 44 = 26$.

PROCEDURE TO SUBMIT A BID

In the upper middle part of the screen you see how you can make your decision in a phase. The cursor on the bar reflects the drop out price that you are willing to submit. By pushing the 'right arrow' key on your keyboard, you can increase your drop out price and by pushing the 'left arrow' key you can decrease your drop out price. Alternatively, you can use the mouse to drag the slider to your preferred drop out price. Once you are satisfied with your drop out price, you push the 'CONFIRMATION' button. Then you will be asked whether you are sure. If you answer 'NO' then you get the possibility to reconsider your drop out price. Once you answer 'YES' your decision is final. In the upper left part of the screen you see the good you want listed after 'Type'. You also see the balance of your total earnings listed after 'Earnings'.

QUESTION ABOUT THE PRICE OF A GOOD

Assume that in the second phase your drop out price equals 61, while the drop out prices of the other five remaining bidders equal 23, 35, 47, 49 and 55. What is the price that you will have to pay for your good?

[Answer: 55. The computer raises the price until the level where all but one participant have dropped out. This way the price will be equal to the second highest submitted drop out price, which in the example is 55.]

FINAL PAYOFFS

When a period is over the next one will begin. Here each participant will be assigned a new good to value and that value will be randomly determined. Hence, the person who values your good in this period will probably not be the same one who valued it in the previous period—that person will be determined randomly in each period. The rules for this period will be the same as those before it and the final payoff you receive at the end of the experiment will be equal to the sum of what you have earned in all periods plus the starting capital. There will be a total of 16 periods in the experiment.

END

You have reached the end of the instructions. If you want to read some parts of the instructions again, push the button BACK. When you are ready, push the button READY. When all participants have pushed READY, the experiment will start. When the experiment has started, you will NOT be able to return to these instructions. Before the experiment is started, a summary of the instructions will be handed out and a practice period will be carried out. Your earnings during the practice period will NOT be added to your total earnings.

If you still have questions, please raise your hand!

References

Armantier, O., Treich, N., 2005. Overbidding in independent private-values auctions and misperception of probabilities. Working paper. Université de Montréal, Montréal, Canada.

Ashenfelter, O., Genesove, D., 1992. Testing for price anomalies in real-estate auctions. Amer. Econ. Rev. 82 (2), 501–505.

Burguet, R., 2005. The condominium problem: Auctions for substitutes. Rev. Econ. Design 9 (2), 73–90.

Goeree, J.K., Offerman, T., 2003. Winner's curse without overbidding. Europ. Econ. Rev. 47 (4), 625–644.

Goeree, J.K., Plott, C.R., Wooders, J., 2004. Bidders' choice auctions: Raising revenues through the right to choose. J. Europ. Econ. Assoc. 2 (2–3), 504–515.

Isaac, M.R., Walker, J.M., 1985. Information and conspiracy in sealed bid auctions. J. Econ. Behav. Organ. 6 (2), 139–159.

Menezes, F.M., Monteiro, P.K., 1998. Simultaneous pooled auctions. J. Real Estate Finance Econ. 17 (3), 219–232.

Milgrom, P.R., Weber, R.J., 2000. A theory of auctions and competitive bidding, II. In: Klemperer, P. (Ed.), The Economic Theory of Auctions. Edward Elgar.

Neugebauer, T., Selten, R., 2003. Individual behavior of first-price sealed-bid auctions: The importance of information feedback in experimental markets. Working paper. University of Bonn, Bonn, Germany.

Ockenfels, A., Selten, R., 2005. Impulse balance equilibrium and feedback in first-price auctions. Games Econ. Behav. 51, 155–170.

Riley, J.G., Samuelson, W.F., 1981. Optimal auctions. Amer. Econ. Rev. 71 (3), 381–392.

Salmon, T.C., Iachini, M., in press. Risk aversion vs. loss aversion in pooled auctions. Games Econ. Behav.