The Effects of Precedent on Arbitration

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This paper presents a new view of the arbitration process in which the arbitrator is depicted as a random device to generate arbitration decisions. The conflicting parties must decide whether to send their dispute to the arbitrator, based on their subjective probability beliefs concerning the arbitration's outcome. If their beliefs are "sufficiently divergent," we can expect both to agree to arbitration. As similar disputes are arbitrated and precedent is set, however, these divergent beliefs can be expected to vanish. Using some game theoretical results of Aumann and Rosenthal, we demonstrate that such convergence will ruin the incentive to arbitrate for conflicts that are zero (or constant) sum in nature, and for games that are non-constant sum but "best responsive equivalent" to zero sum games. It will not do so in general for conflicts that are non-zero sum. We then examine the welfare implications of these results and point out a paradox that arises in this context.

The idea of submitting disputes to arbitration is a very old one, dating as far back as biblical sources such as King Solomon's settlement of a custody dispute. In such instances the arbitrator is depicted as some wise agent who is uniquely capable of arriving at a fair or truthful solution to the conflict at hand. In this paper we treat the arbitration process in quite a different (game theoretical) light. We assume that when two parties are in a conflict they have three options: (1) they can settle their dispute privately by making some mutually agreeable but nonbinding arrangement, i.e., jointly randomize their strategies; (2) they can continue the conflict and receive their noncooperative payoffs; or (3) they can send the dispute to an arbitrator and agree to

AUTHOR'S NOTE: I would like to thank Robert Rosenthal for his help in clarifying my ideas. In addition, I would like to thank the Office of Naval Research whose partial support allowed me to work on this paper. All of the errors and views contained in this paper are, of course, mine alone.

JOURNAL OF CONFLICT RESOLUTION, VOl. 22 No. 4, December 1978

behave in the future in a manner that is consistent with his ruling, i.e., they can correlate their future actions or strategy choices to his decision. However, the arbitrator's eventual decision is not known with certainty by either party so that whether or not the parties agree to arbitration will depend upon their individual subjective probability beliefs about his decision. Consequently, we will depict the arbitration process as a random device which generates decisions or rulings and over which the conflicting parties have differing subjective probability beliefs.

Now, if this view of the arbitration process were correct, we would expect only those disputes to be arbitrated in which the conflicting parties' subjective probability beliefs were "sufficiently divergent" to make each side believe that he had a good chance of "winning." As similar conflicts are repeatedly arbitrated and precedent is set, however, these divergent probability beliefs converge so that one commonly held set of objective beliefs emerges. Precedent is then an informational device which gives parties to arbitration and litigation cases a better basis upon which to evaluate the likelihood of various outcomes.[1] Will such convergent beliefs ruin the incentive to arbitrate? In this paper we will show that in conflicts that are zero sum or constant sum in nature, precedent will indeed ruin the incentive to arbitrate, by destroying the divergence in the conflicting parties' probability beliefs about the outcome of the arbitration. Since the effect of arbitration in such disputes is purely redistributive, if the process is costly, the fact that precedent ruins the incentive to arbitrate can be judged as unequivocally beneficial. For conflicts that are not constant sum or zero sum however, we will see that precedent does not ruin the incentive to arbitrate. In other words, even when both parties agree on the likelihood of the various outcomes of the arbitration, they will still *both* want to have the conflict arbitrated. This is desirable since in these cases arbitration increases utility and does not merely redistribute it. Finally, we will look at the impact of precedent on what we will call "self policing correlated arbitration schemes" and, by using some results of Aumann (1974), and Rosenthal (1974), show the effect of precedent on such schemes. Self policing schemes, which will be defined as schemes that require no binding agreement to enforce them, are particularly important in cases where it is prohibitively costly to enforce compliance

1. Precedent also has many of the qualities of a public good since its use by one arbitrator (or judge) does not preclude its use by another, and if there is "equal justice for all" no one can be excluded from its use. This aspect of precedent may be a useful one to explore.

with an arbitrator's decision. Throughout the paper we shall analyze the consequences of our results for economic welfare. Finally, the theoretical results used in this paper can be found in the extremely interesting articles of Aumann (1974) and Rosenthal (1974). Our paper is an application of their results to a problem of empirical significance.

I

BINDING ARBITRATION IN CONSTANT-SUM CONFLICTS

Consider the following conflict depicted as 2 x 2 game:

Matrix 1

	1	2
1	7,3	13,–3
2	9,1	10½, –½

Notice that the sum of the entries in each element of the matrix is 10 and that the pure strategy saddle point is circled and contains the payoff (9,1). Being constant sum, the interests of the two parties are diametrically opposed and the situation is "strictly competitive." Consequently, in this conflict there can be no "private settlement" other than the one that awards 9 to player 1 and 1 to player 2.

To see this, consider Figure 1 depicting the possible payoffs in the game.

Here we see the point (9,1) representing the value of the game. Any point between the point (13,–3) and (7,3) can be achieved by some jointly randomized strategy agreed to by the players. Such a point would represent a "private settlement" of the conflict. However, because the conflict is constant sum, such a private settlement is obviously impossible. For, if the private settlement determined a point below and to the right of point (9,1), party 2 would not agree to it, while if it determined a point above and to the left of (9,1), party 1 would not agree to it. Therefore, conventional game theory concludes that the only outcome that makes sense for this game is the saddle point value (9,1), and it is here that the theory rests.

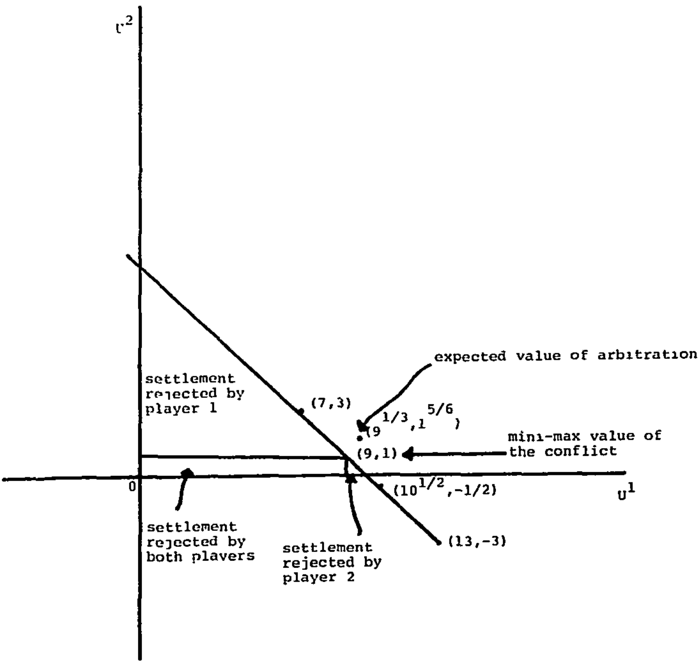

Figure 1.

Recently, Robert Aumann (1974) has expanded the possibilities available to our players in a very simple yet dramatic way and has actually found a way for the players in constant- or zero-sum conflicts to cooperate. The idea is simple. Assume that there exists an event D, totally exogenous to the game under investigation, and that player 1 feels that the probability of D occurring is 1/3 while the probability of its not occurring is 2/3. Assume that player 2 has just the opposite belief. If this is so, then the players could enter in a binding way into the following agreement: if D occurs they will both play strategy 1, whereas if D does not occur they will both play strategy 2. If they do this, then the expected payoff to player 1 from this "subjectively randomized strategy" will be $9^{1/3}$, while the expected payoff to player 2 is $1^{5/6}$, which in terms of expected utility is a result preferable to (9,1).

How does this result relate to arbitration? The answer is obvious. Consider the situation faced by our two parties. Clearly they have three choices: (1) to use their mini-max strategy and play the game noncooperatively, (2) to settle privately by agreeing to some jointly randomized strategy, or (3) to send the case to arbitration.

Now, as we have seen, private settlement in such cases is impossible. However, if party 2 believes that the arbitration decision will be in his favor with a probability of 2/3 and against him with a probability of 1/3 and party 1 holds just the opposite beliefs, then the following arbitration scheme is possible: if the arbitrator rules for party 2, both parties must choose strategy 2, whereas if he rules against party 2, both parties must choose strategy 1. We will call such a scheme a "correlated arbitration scheme" (CAS). It depicts the arbitrator's decision as some exogenous random device (a "public roulette," to use Aumann's term). Basically all the scheme says is that if party 2 "wins" the arbitration, both will choose strategy 1 (the outcome of which is favorable to party 2), whereas if party 1 "wins" both will choose strategy 2 (the outcome of which is favorable to party 1). The scheme merely defines what winning and losing mean, given the probabilities defined over the arbitrator's decision.

There is nothing artificial about this view of the arbitration process, since in all arbitration the actions of the parties are geared to or correlated with the arbitrator's decision in a binding manner. Here the arbitrated expected payoffs are ($9^{1/3}$, $1^{5/6}$), and since they are Pareto superior to the value of the game, we can expect both parties to opt for the arbitration.

Notice, however, that the beliefs of our two parties are illusory since the expected payoffs from arbitration add up to $11^{1/6}$, and such a payoff is physically impossible since the sum of the payoffs in our game can only be 10. This "arbitration illusion" is responsible for the incidence of arbitration in constant sum conflicts since it leads both parties to believe that they are better off arbitrating than playing noncooperatively and accepting the value of the game. Notice also that the arbitration scheme requires a binding agreement to enforce it and that in the absence of such an agreement each party has an incentive to deviate.

Finally, the arbitration process pictured above can be summarized by the following scenario. First, the conflict is depicted as a two person bi-matrix game. Then, each party bargains over the CAS to be used which is, in essence, bargaining over what "winning" and "losing" the arbitration mean. Finally, if any CAS can be found that promises each party an expected payoff greater than its payoff under the value of the game, the parties bindingly agree to follow the dictates of that scheme, no matter what the arbitrator's decision is.

PRECEDENT AND THE INCIDENCE OF ARBITRATION IN CONSTANT SUM CONFLICTS

As we have just seen, if the probability beliefs of the parties in conflict situations are "significantly divergent," both parties will agree to submit their dispute to binding arbitration because the expected payoff from the arbitration is greater for both parties than their expected payoffs from playing the game. In our example, using the strategy pairs (1, 1) and (2, 2), player 1 must think that his chances of winning are strictly greater than 4/7 while player 2 must think that his chances of winning are strictly greater than 3/7 in order for both of them to submit their conflict to arbitration. Such beliefs are inconsistent, however, since a true probability estimate can never give one party a strictly greater than 4/7 chance of winning and simultaneously give the other party a strictly greater than 3/7 chance of winning. Therefore, if the effect of precedent in our example is to make the probability beliefs of the parties converge, no arbitration would be agreed to, since it would have to be "mutually beneficial," i.e., to give each party an expected payoff strictly greater than that he would receive from the value of the game. In the present case this is clearly impossible. To formalize this result, consider the following proposition.

Proposition 1: No arbitration is possible in constant sum (and therefore zero sum) conflicts with precedent (i.e., in which precedent has determined convergent probability beliefs among the parties).

Proof: The proof follows easily from the definition of a constant sum game and the fact that the arbitration scheme must give each party *strictly* more than the value of the game in order for them to want to arbitrate. To see this, let $\bar{x} = (\bar{x}_1, \ldots \bar{x}_n)$ and $\bar{y} = (\bar{y}_1, \ldots \bar{y}_m)$ be two n-tuples of mixed strategies, one for player 1 and one for player 2 in the constant sum game (A,B), and let $M = (M_1 \ldots, M_\ell)$ be the set of possible arbitration decisions with probabilities $p_1, \ldots p_\ell$ respectively. Assume that these probability estimates are commonly held by both players and let $a = \{\alpha_{ij}, k-\alpha_{ij}\}$, $i=1, \ldots, n$, $j=1, \ldots, m$ be the set of expected payoffs that result when player 1 uses strategy i and player 2 uses strategy j.

Consider the following CAS: if M_1 is the arbitration decision, then player 1 will choose $\bar{x}_i$ and player 2 will choose $\bar{y}_j$. If the arbitration decision is M_2, then player 1 will choose $\bar{x}_\ell$ and player 2 will choose $\bar{y}_m$, etc.

If such a scheme is agreed to by both parties, it must give each of them an expected payoff which is greater than the value of the game $v = (v_1, v_2)$ (where $v_1 + v_2 = k$). This is not possible, however, since in the scheme outlined above, the expected total payoff of the two parties is

$$E(\Pi) = p_1(\alpha_{ij}+k-\alpha_{ij}) + p_2(\alpha_{\ell m}+k-\alpha_{\ell m}) + \ldots\ldots\ldots\ldots + p_\ell(\alpha_{zw}+k-\alpha_{zw})$$

$$= \sum_{i=1}^{\ell} p_i k = k$$

and there is no way to give both parties strictly more than they receive from the value of the game. Therefore, no arbitration scheme can be agreed to. QED.

SOME WELFARE ASPECTS OF PRECEDENT IN TWO PERSON CONSTANT-SUM CONFLICTS

The question that logically arises is whether precedent is socially beneficial for constant-sum conflicts. The answer is unequivocally yes if the arbitration process has any cost associated with it at all. The reason for this is simple. In constant-sum conflicts, all outcomes are Pareto optimal. Therefore, the effect of arbitration is strictly distributive. Consequently, if the process itself is costly, it would be wasteful to arbitrate such cases and to the extent that precedent eliminates arbitration in such cases it must be considered beneficial.

To see this more clearly, consider Figure 1. There, in the example given in the section "Binding Arbitration in Constant-Sum Conflicts," with divergent probability beliefs, the expected outcomes of the arbitration for the two parties was the point $(9^{1/3}, 1^{5/6})$. As was said before, this point is illusory since the only physically possible points associated with the arbitration are the points on the line between the points

(7, 3) and (13, −3), all of which add up to 10. However, if arbitration costs were less than $1^{1/6}$, there would exist a way to charge the parties for the costs of arbitration so that even in the face of these costs both would agree to arbitrate. This is wasteful, however, and occurs because of the "arbitration illusion" referred to above. To the extent that precedent destroys this illusion, it destroys the incentive to arbitrate, and in constant-sum conflicts this is beneficial (at least in terms of potential welfare).

ARBITRATION IN NON-CONSTANT SUM CONFLICTS

When we turn to non-constant conflicts, the situation is more complex. Consider the following conflict situation:

Matrix 2

	1	2
1	11,11	7,12
2	12,7	5,5

This is a linear transformation of Aumann's (1974) example 2.7.

This game has three Nash equilibria, two in pure strategies with payoffs (12,7) and (7,12) and one in mixed strategies with payoffs ($9^{2/3}$,$9^{2/3}$). In this conflict our two parties again have three options: (1) to play non-cooperatively, (2) to settle privately, but non-bindingly, or (3) to bindingly arbitrate.

If they play noncooperatively, their payoffs will be one of the three Nash equilibrium payoffs. By settling privately, they can jointly randomize their strategies and achieve any payoff in the convex hull of the set of Nash equilibrium payoffs (see Harsanyi and Selten, 1972) without making binding agreements.[2] When it comes to arbitration, however, we will see that they can achieve expected payoffs which are outside of the convex hull of Nash equilibrium payoffs even if both parties totally agree on the probabilities associated with the arbitration's outcomes. It is for this reason that both parties agree to arbitrate. Consequently, for non-constant-sum conflicts, divergent probabilities are not necessary in order for both parties to desire to arbitrate.

2. This result relies on each Nash equilibrium being a "strong" equilibrium point, which in our example they are. Also, we rule out, "private" agreements being binding because bindingness implies some authority (the state or courts) to enforce it which is ruled out in "private" agreements.

To demonstrate this point, let there be four possible outcomes of the arbitration: (1) party 1 is found in the wrong (W) and must pay damages to party 2 (D), in which case party 2 "wins," (2) party 1 is found in the wrong (W) but does not have to pay damages to party 2 (ND), the "compromise" solution (3) party 1 is found not to be in the wrong (NW) and does not have to pay damages (ND), in which case party 1 "wins," and (4) finally party 1 can be found to not be in the wrong (NW) but still be forced to pay damages (D), an outcome we are ruling out as impossible. Let the parties assign the following extremely divergent probabilities to these outcomes:

Matrix 3

Party 1		ND	D
	W	0	0
	NW	1	0

Matrix 4

Party 2		ND	D
	W	0	1
	NW	0	0

Now the following CAS can be suggested, to which the parties are bound to comply: If party 1 is found in the wrong but no damages are requested, both parties must choose strategy 1. If party 1 is found in the wrong and damages are demanded, party 1 must choose strategy 1 and party 2 must choose strategy 2. Finally, if party 1 is found not in the wrong and no damages are requested, party 1 must choose strategy 2 and party 2 must choose strategy 1.

Because of the extremely divergent beliefs of the parties, the expected payoff for this scheme is (12,12) a payoff that not only is outside of the convex hull of the Nash equilibrium payoffs but is also Pareto superior to the outcome (11,11), which can only be achieved by private binding agreement.

However, in this case even if both parties agree on the probabilities of the various arbitration outcomes (i.e., even in the face of precedent), both might agree to send their case to arbitration since the expected outcome would be outside of the convex hull of Nash equilibria, the set of points they can guarantee themselves by "private settlement." To see this, consider the same CAS outlined above, but assume the following set of commonly held objective probability beliefs:

Matrix 5

	ND	D
W	1/3	1/3
NW	1/3	0

The expected payoff for the players for this scheme is (10, 10), which is outside the convex hull of Nash equilibrium payoffs. Consequently, arbitration offers an outcome for the parties that they could not achieve by any private settlement. In addition, notice that there does not need to be a divergence in probability beliefs in order for this result to occur. In fact, party 1 agrees with party 2 that party 1 has a two-thirds chance of being found in the wrong and still has incentive to agree to arbitrate.

The reason why arbitration seems so profitable is that by correlating their strategy choices with the arbitrator's decision, both parties can guarantee themselves that they can avoid the mutually destructive outcome (5, 5), and this is something that they cannot do without the arbitrator, using strategies which are only jointly randomized.

SOME WELFARE CONSIDERATIONS

In constant-sum disputes, we saw that precedent was beneficial because it ruined the incentive to arbitrate; and since arbitration in constant-sum disputes is purely redistributive, this was seen as being unequivocally good. In non-constant-sum disputes, it would not be beneficial if precedent ruined the incentive to arbitrate, since arbitration in these disputes increases utility and does not merely redistribute it. We would want these disputes arbitrated so as to avoid suboptimal Nash equilibrium payoffs. However, if the arbitration process is costly, we would not want all disputes arbitrated since the expected benefits from arbitration may be less than the costs. Precedent insures that this will not happen. To see this, consider Figure 2, which is a geometric representation of the conflict depicted in Matrix 2.

Here, the shaded area represents the convex hull of Nash equilibrium payoffs while the polyhedron containing it depicts all of the payoffs that are possible under arbitration. Notice the maximum payoff achievable through arbitration is (12, 12), which occurs when the players have extremely divergent beliefs (matrices 3 and 4). Clearly, the polyhedron contains points such as (10, 10), the expected outcome of the CAS described above, which are outside of the convex hull of Nash equilibria but still feasible through arbitration. Assume that the arbitration process is costly and that each party will be charged 6 1/2 each to pay for it. If this is true, then this dispute should clearly not be arbitrated since the maximum possible joint payoff is (11, 11), whose sum is 22 and when the 13 unit cost of arbitration is subtracted from 22, the remaining amount of utility available to the two players is

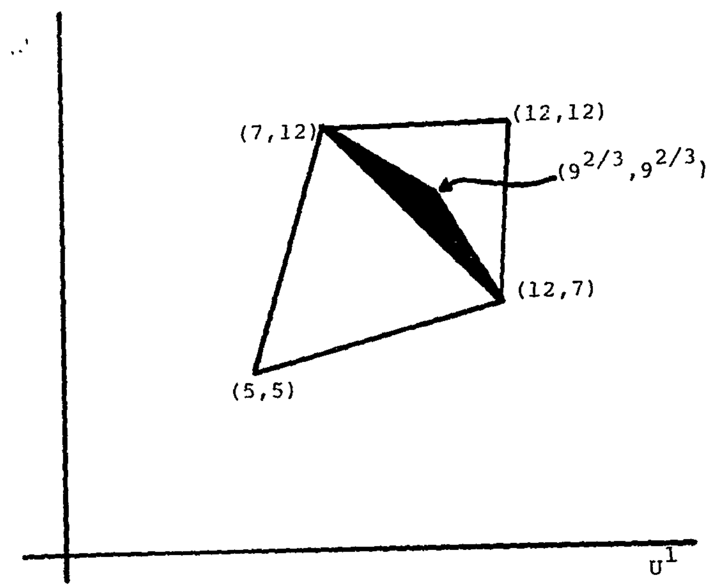

Figure 2.

only 9 which is less than the 10 units that our two players can guarantee themselves if they play noncooperatively and achieve an equilibrium payoff of (5, 5). However, if the parties' beliefs are extremely divergent, as we depicted them above (matrices 3 and 4), then the "arbitration illusion" leads them to think that a payoff of (12, 12) is their expected payoff, and with this payoff they will be willing to jointly send the case to arbitration because their illusory expected net gains are positive. Precedent, by eliminating these types of illusions, guarantees that only disputes whose true expected net payoffs are positive are actually arbitrated. Therefore, while precedent in non-zero-sum conflicts does not ruin the incentive to arbitrate, it does guarantee that only those cases whose true net benefits (benefits minus costs) are positive are arbitrated.

II

The arbitration schemes outlined in section I which were called "correlated arbitration schemes" all relied on binding agreements among the parties in order to ensure that they would actually follow the correlated strategies specified by the scheme. However, in certain disputes it may be impossible or prohibitively costly to detect whether a party actually did follow his agreed strategy. For instance, let us assume that the Soviet Union and the United States decide not to negotiate another Strategic Arms Limitation Treaty but rather send their dispute to some neutral arbitrator and follow his decision. Here, the correlated strategies may involve the destruction of missiles or a cessation in the development of arms systems. However, as experience tells us, there does not exist a feasible way to police these schemes and cheating is bound to occur. What is needed is a self-policing scheme or one in which external enforcement is not necessary because the arbitration scheme is self-enforcing. In this section we will turn our attention to such schemes and discuss the effect of precedent on them.

SELF-POLICING ARBITRATION SCHEMES

The idea of devising self-policing arbitration schemes can be found in Aumann (1974). It is only used to inform partially the parties in the conflict of the arbitrator's decision and to correlate their strategies with this partial information. To illustrate, let us look at the CAS depicted in Section I—which required binding agreements in order to be followed. In that case the outcomes of the arbitration were two-element events in which one element was chosen from the set W = (W, NW) and one was chosen from the set D = (D, ND).

Consider the following arbitration scheme. Party 1 is told whether he must pay damages to party 2 but not whether he has been found in the wrong. Party 2 is told whether party 1 was found in the wrong but not whether damages are to be paid. If party 1 is told that damages must be paid he chooses strategy 2, but if he is told no damages he chooses strategy 1. Likewise, if party 2 is told that party 1 was found in the wrong he plays strategy 1, whereas if he is told that 1 was found not to be in the wrong he plays strategy 2. Assume that both parties commonly hold the following probability beliefs about the arbitration's outcome:

	ND	D
W	1/3	1/3
NW	1/3	0

The expected payoff from this scheme is again (10, 10). In addition, however, this scheme is self-policing, since whatever the decision of the arbitrator, given their partial information, both parties will find it more beneficial in terms of expected utility to follow the prescribed behavior of the arbitration scheme than to deviate. The scheme is then a "self-policing correlated arbitration scheme" (SPCAS).

To formalize this, let (A,B) be a bi-matrix game representing the conflict to be arbitrated and let $C = (C_1, \ldots, C_k)$ and $D = (D_1, \ldots, D_\ell)$ represent two partitions of the arbitrator's outcome space. In addition, these two partitions will represent two partitions of a probability space (Ω,β,P). The actual income of the arbitration will be a joint event $(D_j \cap C_i)$, $D_j \epsilon D$, $C_i \epsilon C$, and $P(D_j \cap C_i)$ will represent the probability of that event. Let $\bar{y} = (\bar{y}_1, \ldots, \bar{y}_\ell)$ and $x = (x_1, \ldots, x_k)$ be ℓ and k mixed strategies for players 2 and 1 respectively. Then $(\bar{x}/C, \bar{y}/D) = (\bar{x}^1/C_1, \ldots, \bar{x}^k/C_k, \bar{y}^1/D_1, \ldots, \bar{y}\ /D_\ell)$ is a partially correlated arbitration scheme and if the scheme is to be a SPCAS the following must hold:

$$\bar{x}^{iT} A \sum_{j=1}^{\ell} P(D_j \cap C_i)\, \bar{y}^j \geqslant x^T A \sum_{j=1}^{\ell} P(D_j \cap C_i)\, \bar{y}^j$$

for all mixed strategies x for player 1 and for all i ϵ {1, . . . , K}

$$\sum_{i=1}^{k} P(C_i \cap D_j)\, \bar{x}^{iT} B \bar{y}^j \geqslant \sum_{i=1}^{k} P(C_i \cap D_j)\, \bar{x}^{iT} B y$$

for all mixed strategies for player 2 and all j ϵ {1, . . . , ℓ .}.

Consequently, in disputes where binding agreements are not possible because of impossibility of enforcement, any arbitration scheme proposed must satisfy two conditions if it is to be acceptable to the parties and the conflict arbitrated. (1) The scheme must be SPCAS. This is necessary because the arbitration process would be meaningless if the parties could not expect each other actually to adhere to it. (2) The expected outcome of the scheme must be strictly outside of the convex hull of Nash equilibrium payoffs (or Pareto superior to the value of the game if the game is constant sum). This is necessary to give the parties a positive incentive to arbitrate.

Any SPCAS that satisfies these two conditions is called "acceptable," and arbitration will occur only when an acceptable arbitration scheme can be found. Otherwise, the parties would either prefer to play noncooperatively, or settle privately by jointly randomizing.

PRECEDENT AND THE INCIDENCE OF ARBITRATION IN SELF-POLICING CORRELATED ARBITRATION SCHEMES

Having described how self-policing arbitration schemes work, we are now in a position to investigate the effect of precedent on them. We will see that whether or not precedent destroys the incentive to arbitrate using a SPCAS depends on the type of conflicts under consideration. In all of our discussion, however, we will assume that precedent determines strictly convergent probability beliefs.

To begin, it should be obvious from our example in the first part of section II that one can construct "self policing correlated arbitration schemes" for some conflicts that are non-constant-sum conflicts, and that such schemes are possible even in the face of precedent. However, for constant-sum conflicts, this is not possible since in these cases precedent always ruins the incentive to arbitrate by eliminating the existence of acceptable SPCAS. The following proposition states this result.

Proposition 2: (Rosenthal): In the face of precedent no "acceptable" SPCAS can be found for conflicts that are constant- or zero-sum conflicts.

Proof: In order for the SPCAS to be acceptable, it must determine an expected value for the players that is strictly greater than their expected value at any NASH equilibrium (x*,y*). To prove our theorem, then, we will show that for any NASH equilibrium all SPCASs will determine an expected value exactly equal to the value at (x*,y*) and consequently cannot be acceptable.

To begin, let (A,−A) represent a zero-sum game. Let $(\bar{x}^1/C_1, \ldots, \bar{x}^k/C_k)$, $(\bar{y}^1/D_1, \ldots, y^\ell/D_\ell)$ be a correlated equilibrium, and let (x^*, y^*) be a NASH equilibrium.

Then,

$$\sum_{i=1}^{k} \bar{x}^{iT} A \sum_{j=1}^{\ell} P(D_j \cap C_i)\, \bar{y}^j \geqslant \sum_{i=1}^{k} x^{*T} A \sum_{j=1}^{\ell} P(D_j \cap C_i) \bar{y}^j$$

$$= x^{*T} A \sum_{j=1}^{\ell} P(D_j) \bar{y}^j \geqslant x^{*T} Ay^*.$$

But,

$$\sum_{i=1}^{k} \bar{x}^{iT} A \sum_{j=1}^{\ell} P(D_j \cap C_i) \bar{y}^j = \sum_{j=1}^{\ell} y^{jT} A^T \sum_{i=1}^{k} P(C_i \cap D_j) \bar{x}^i$$

which is $\leqslant x^{*T} Ay^*$ by a similar argument. Consequently, the payoffs for any SPCAS must be exactly equal to the payoffs at that NASH equilibrium and such a SPCAS cannot be acceptable. QED.

Corollary: In the above proof,

$$\sum_{i=1}^{k} P(C_i) \bar{x}^i \text{ and } \sum_{i=1}^{\ell} P(D_j) \bar{y}^j$$

are also NASH equilibrium.

From this discussion it would appear that the effect of precedent on self-policing correlated arbitration schemes depends on whether the conflict is "strictly competitive" or not. As can be shown (see Rosenthal), this is not quite the proper distinction to make. First of all, there exist conflicts which, while not "strictly competitive," have very competitive natures nevertheless, yet do admit "acceptable" SPCAS. Such are the classes of games which are "almost strictly competitive" games as well as those games which are "order equivalent" to zero-sum games (while not zero sum themselves). More interesting, however, is the fact that there exists a class of conflicts which are non-constant-sum but for which, in the face of precedent, no SPCAS can be found. This is the class of games which are "best response equivalent" to zero-sum games.[3] Here, even though the conflicts may be non-constant sum

3 A conflict (A,B) is "best response equivalent" to a zero-sum game (A',B') of equal dimension if under some ordering of the pure strategies in both games the set of strategies associated with the set of best responses to any mixed strategy in either game is precisely the set of best responses to the strategy in the other game. Such games need not be zero or constant sum.

so that the parties have incentives to cooperate in order to avoid mutually destructive outcomes, there do not exist SPCASs which determine outcomes outside of the convex hull of Nash equilibria. Consequently, for these conflicts there cannot exist any "acceptable" SPCAS. This can be summarized by the following proposition.

Proposition 3: (Rosenthal Theorem 1): If (A,B) is a conflict that is best response equivalent to a zero sum game, then in the face of precedent no acceptable SPCAS exists.

Proof: Here, what we will prove is that any SPCAS will determine payoffs whose components are exactly equal to the payoff at some NASH equilibrium and can therefore not be acceptable. Let $(\bar{x}^1 C_1, \ldots, \bar{x}^k/C_k), (\bar{y}^1/D_1, \ldots, \bar{y}^\ell/D_\ell)$ be a SPCAS, and let (x^*,y^*) be any NASH equilibrium for (A,B). From the proof of Proposition 2 it is clear that x^* is a best response to $\sum_i P(D_j \cap C_i)\bar{y}^j$ whenever $P(C_i) > 0$. Hence,

$$\sum_i \bar{x}^{iT} A \sum_j P(D_j \cap C_i)\bar{y}^j = \sum_i x^{*T} A \sum_j P(D_j \cap C_i)\bar{y}^j =$$

$$x^{*T} A \sum_j P(D_j)\bar{y}^j.$$

By corrollary 1, $(x^*, \sum_j P(D_j)\bar{y}^j)$ is a NASH equilibrium. Similarly,

$$\sum_j \bar{y}^{jT} B^T \sum_i P(C_i \cap D_j)\bar{x}^i = y^{*T} B^T \sum_i P(C_i)\bar{x}^i$$

and $(\sum_i P(C_i)x^i, y^*)$ is a NASH equilibrium. However, since the conflict (A,B) is best response equivalent to a zero-sum game, the equilibria in (A,B) must be interchangeable. Therefore, the payoffs to the players must be payoffs associated to some NASH equilibrium and consequently cannot be acceptable since that fact prevents them from being strictly greater than the NASH equilibrium payoffs. QED.

Notice that this result holds only in those circumstances in which precedent is so strong that it leads to totally convergent probability estimates by the parties. If the probabilities diverge, then such a divergence might lead the parties to arbitrate.

SOME WELFARE CONSIDERATIONS— THE PARADOX OF BETTER INFORMATION

Clearly all that we have said in Section I about the beneficial aspects of precedent for binding arbitration schemes holds for self-policing schemes as well. However, in self-policing schemes we have seen that in the face of precedent conflicts that are not constant sum, but rather "best-response-equivalent" to constant-sum games, will never be arbitrated. This may not be beneficial, however, since the effects of arbitration in these disputes is not purely redistributive, but because of their possible non-constant-sum nature arbitration may actually increase utility. Therefore, to the extent that precedent ruins the incentive to arbitrate these diputes, it may be detrimental.

This result is interesting for the following reason. In conflicts that are "best-response-equivalent" to zero-sum or constant-sum-games, our conflicting parties are faced with the decision of whether or not they want their conflict arbitrated. Economic theory tells us that if individual agents must make decisions the quality of their decisions (however measured) should monotonically increase with the quality of the information they obtain, especially if that information is free. Since precedent is a public good which is supplied free to the user, if offers better information upon which to make decisions and should therefore increase welfare. In the case of disputes that are "best-response-equivalent" to zero-sum or constant-sum-games, just the opposite is true. Precedent, by furnishing "better" information to the conflicting parties leads them *not* to arbitrate cases which should be arbitrated. Put another way, in cases that are "best-response-equivalent" to zero-sum or constant-sum games, an arbitration illusion is needed to send the case to arbitration and to the extent that precedent destroys this illusion, it diminishes social welfare.

AN EXAMPLE

Before we conclude, it may be useful to present one example which will illustrate the thrust of proposition 3 (and, as a byproduct, of proposition 2 also). To do this, consider the following two matrices:

	β_1	β_2	β_3
α_1	4,–4	6,–6	7,–7
α_2	9,–9	4,–4	10,–10
α_3	0,0	0,0	0,0

Matrix 6

	β_1	β_2	β_3
α_1	4,–4	6,–6	7,–8
α_2	9,–9	4,–4	10,–5
α_3	2,0	3,1	1,0

Matrix 7

Clearly, Matrix 7 is best response equivalent (BRE) to Matrix 6, since in both cases α_3 and β_3 are dominated strategies and therefore would always receive zero weight in any best-reply strategy. Once they are eliminated, the remaining 2 x 2 games are identical, hence they are BRE.

Assume that the conflicting parties are contemplating taking this dispute to an arbitrator who can offer them a nonbinding settlement. Assume that the outcome of the arbitration can be partitioned in the following manner $[C_1,C_2)(D_1,D_2)]$ and that if C_1 is found by the arbitrator, player 1 will be asked to play strategy α_1 and if C_2 is found he will be asked to play strategy α_2. Also, assume that if D_1 is found player 2 will be asked to play strategy β_1 and if D_2 is found he will be asked to play strategy β_2. Finally, assume that precedent in such conflicts is so strong that all players agree that $P(C_1) = 5/7$, $P(C_2) = 2/7$, $P(D_1) = 2/7$, $P(D_2) = 5/7$ are the relevant probabilities so that both players agree that the joint probability of C_i and D_j occurring are

	D_1	C_2
C_1	$P(C_1 \cap D_1)=\frac{10}{49}$	$P(C_1 \cap D_2)=\frac{25}{49}$
C_2	$P(C_2 \cap D_1)=\frac{4}{49}$	$P(C_2 \cap D_2)=\frac{10}{49}$

and that player 1 is informed only of the outcomes C_1 and C_2 while player 2 is informed only of the outcomes D_1 and D_2. This arbitration scheme is consequently an equilibrium SPCAS since neither player has any positive incentive to deviate from his assigned strategy once he is informed by the arbitrator of the information he is privileged to. The value of the scheme is 5.42 for player 1 and −5.42 for player 2. The scheme is not "acceptable," however, since the mixed strategies x = (5/7, 2/7) for player 1 and y = (2/7, 5/7) for player 2 are Nash equilibria for the game portrayed in Matrix 7 and the expected value for player 1 at the equilibrium is 5.42 while the expected value for player 2 is −5.42. Consequently, arbitration does not yield a payoff that is outside the set of Nash equilibrium payoffs for the game and, hence, there does not exist a positive incentive to arbitrate. The point of proposition 3 is that this type of result is always true for games that are best-response-equivalent to zero-sum games. That is, the probabilities dictated by the scheme, if employed by the players in isolation, would determine Nash equilibrium payoffs equivalent to the payoffs defined by the scheme, if the scheme is an equilibrium SPCAS. Consequently, they cannot be outside of the set of Nash equilibrium payoffs and hence, not acceptable. Actually since α_3 and β_3 are dominated, this result follows from the fact that Matrices 6 and 7, after α_3 and β_3 are removed, are identical zero-sum games. Therefore, this example may illustrate Proposition 2 better than Proposition 3. Yet even though Matrix 7 is BRE to Matrix 6 in a degenerate manner, the thrust of the argument still applies.

CONCLUSION

As we have seen, precedent in arbitration is, in most cases, an unequivocally good thing. In all constant-sum conflict it prevents wasteful arbitration from occurring. In most non-constant-sum conflicts it allows only those conflicts to be arbitrated whose net gain, after the subtraction of arbitration costs, is positive. In some instances, in conflicts that are best response equivalent to zero sum games, however, we have seen that precedent may ruin the incentive to arbitrate cases that in fact should be arbitrated. However, these are the exception and not the rule. In the final analysis, then, we feel that since arbitration precedent has not as yet been firmly established, its institution would be a beneficial act.

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