Merger Illusions and Externalities

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Introduction

In this note we hope to accomplish two objectives. *First,* to alert policymakers to the importance of historical considerations when contemplating corrective taxes as a policy used to handle externalities. *Second,* to elucidate once more the usefulness of the core concept in the study of economies containing externalities.[1]

The "Tree Game"

Let us consider the following community consisting of three agents or players situated as follows:

Player C	Player A	Player B

Assume that the economy faces the following problems: Player A bestows a beneficial externality on Player B and a harmful externality on Player C. (The reader may think of Player A as planting trees to grow apples, Player B as a bee keeper, and Player C as the owner of a swimming pool which is polluted by falling tree leaves.) Now obviously because of the externality, the problem faced is that Player B wants Player A to increase his production (grow more trees), but Player C wants the production reduced (i.e., he wants some trees to be cut down).

Let us say that Player A, the apple grower, can sell his apples in a perfectly competitive apple market and that the optimal number of trees he would plant if acting alone is $\bar{t}$.[2] In addition, assume that Player B is willing to offer Player A a payment to plant a number $t^* > \bar{t}$ trees. Similarly, Player C is willing to pay for a reduction of the number of trees planted by A from $\bar{t}$ to $\tilde{t} < \bar{t}$. It follows, then, that Player B, acting alone can guarantee himself only the utility associated with $\tilde{t}$ trees, while Player C, if acting alone, can guarantee himself only the utility associated with t^* trees. For the sake of argument, let us assume all of the proper convexity assumptions hold, and that B is willing to offer more to A for the addition of the $\bar{t} + 1$st tree than C is willing to offer for the destruction of the $\bar{t}$th. Also assume that all the players' utility functions are linear in money so that side payments can be affected.

With these assumptions, the characteristic function[3] of the "tree game" is:

$$
\begin{aligned}
V(\mathrm{A}) &= U_a(\bar{t}) \\
V(\mathrm{B}) &= U_b(\tilde{t}) \\
V(\mathrm{C}) &= U_c(t^*) \\
V(\mathrm{AB}) &= \max_t \,[U_a(t) + U_b(t)] \\
V(\mathrm{AC}) &= \max_t \,[U_a(t) + U_c(t)] \\
V(\mathrm{ABC}) &= \max_t \,[U_a(t) + U_b(t) + U_c(t)]\,.
\end{aligned}
$$

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[1] There have been a number of important papers dealing with externalities but they are mathematically very sophisticated and consequently not very accessable to many economists.

[2] We will assume that trees grow instantly and produce a fixed number of apples and that it is possible to plant a fraction of a tree, etc. Consequently the marginal cost of an apple is the marginal cost of planting that fraction of the tree needed to produce that apple, measured in labor cost, if you wish.

[3] We will assume that the game is played only in tree strategy space since it may be, as Coase indicates, that the true Pareto optimal solution would be to move Player C to the other side of Player B, but we will ignore this. See R. Coase, "The Problem of Social Cost," *The Journal of Law and Economics* (October 1960).

We can prove the existence of a core for this game in which a number of trees $t'' > \bar{t}$ is decided upon and which cannot be improved upon by any coalition of A's, B's and C's.[4]

The Merger Illusion

Now let us assume that instead of side payments, A and B decide to merge and internalize the economies they jointly create. After the merger the economy looks like this:

Player C	Player A–B

In addition, assume that a Pigouvian analyst, unaware of the history of the joint company, A–B, sees only the negative externality placed on C by A–B and decides to tax the merged firm so that it will reduce the number of trees it plants and thereby reduce the level of externality the merged firm imposes on Player C. The question remaining is, of course, how can the situation be an improvement in social welfare after the tax has reduced the number of trees? Obviously it cannot be since we have assumed perfect communication before the merger, and if the newly determined number of trees is optimal, it would have been decided upon in the original bargaining. The problem is, of course, that the policy-maker is taxing the merged firm on exactly that externality which originally created the merger. Consequently, the analyst is suffering from a "merger illusion" which he would not be suffering from if the firms had not merged. In addition, it follows from the definition of the core, that a reduction in the number of trees below t'' cannot determine a utility imputation that dominates any imputation in the core of the original game. Therefore, the reduction below t'' cannot be a welfare improvement since Player B loses more than Player C gains by the reduction. This is not to say that the newly merged firm should not be taxed or forced to reduce the level of some other externality, as long as this externality did not create the grounds for the original merger, assuming perfect communications throughout.

To conclude, when deciding upon a policy for the handling of externalities, we must take care to consider the history of the situation we are observing in order to make sure we do not suffer from any similar illusions. Such considerations may affect substantially both the direction and the magnitude of corrective government action.

Some Observations on the Core Solution

It is interesting to note that some ambiguities arise when we use standard terminology of welfare economics in this analysis. The ambiguity is as follows: a competent economist when looking at our original example recognizes that all possible solutions are on the Pareto optimal locus. That is, the competitive equilibrium has $\bar{t}$ trees, and any addition or subtraction of trees will benefit one person while hurting another. Specifically, when looking at a projection of our three-dimensional utility space onto the two-dimensional utility space (U_b, U_c), the typical analysis would be to view the situation as in Figure 1. In this Figure U_i is the utility of the ith player. Here the "status quo" competitive equilibrium is at $\bar{T}$, and along the Pareto frontier ZZ', the interests of the two participants are diametrically opposed. This projection assumes that Player A has already received a compensation, p, for adjusting the number of trees.

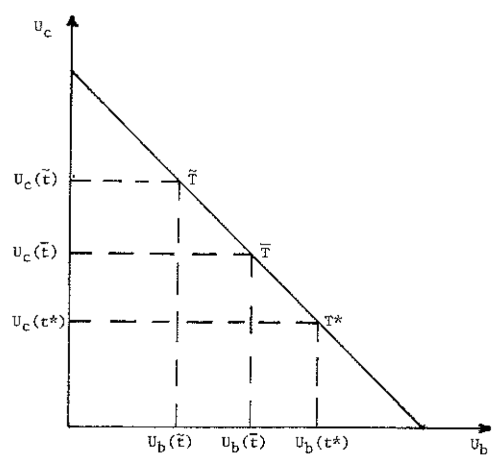

Figure 1

[4] For a detailed discussion of the concept of the core, see L. Telser, "Competition, Collusion, and Game Theory," Chicago, 1972; A. Schotter, "Core Allocations and Competitive Equilibrium–A Survey," *Zeitschrift fur Nationalokonomie,* 33 (1973), pp. 281–315.

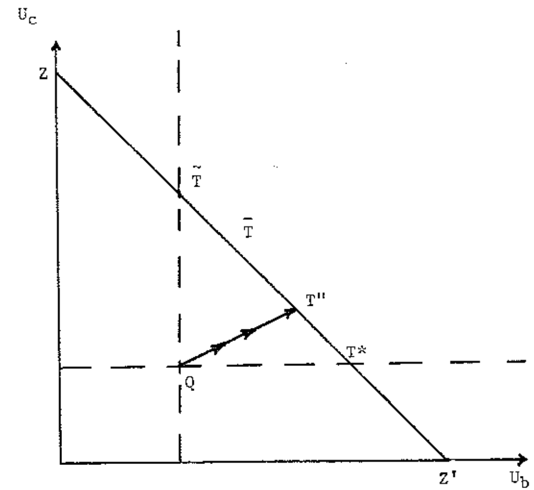

Figure 2

The game theoretical analysis, however, does not start out at point $\bar{T}$. Looking at the characteristic function, we realize that the best that Player B can guarantee himself is the value of the game to him when A and C form a coalition and choose $\tilde{t}$ trees, while the best Player C can do is to accept the value of the game to him when A and B form a coalition and plant t^* trees. This is represented by the point Q in Figure 2, which is inside the Pareto frontier.

The point T'' represents the projection of the core onto this utility space. Notice that if the number of trees planted were less than t'' (represented by a point between T'' and Z), a coalition of A and B could block the associated imputation. If the number of trees planted is greater than t'', represented by a point between T'' and Z', a coalition of A and C could block the associated imputation. In terms of our diagram, the imputation associated with point T'' dominates any point on the line ZZ'.

Furthermore, notice that if Player C was either not present or not affected by the externality, t^* trees would be planted. Therefore, the difference between t^* and t'' measures the strategic influence of this extra-marginal player in the game.[5] In other words, while player C is not able to decrease the number of trees below $\bar{t}$, he does place a limit on the number of trees Player B is able to bribe A into planting.

[5] For a similar result in market games, see L. Shapley and M. Shubik, "The Assignment Game I: The Core," *International Journal of Game Theory,* Vol. 1, July 1972, pp. 111–130, and for auctions see A. Schotter, "Auctioning Bohm-Bawerk's Horses," *International Journal of Game Theory,* Vol. 3, Issue 4, pp. 195–215.