Disadvantageous Syndicates in Public Goods Economies

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The formation of syndicates, unions, or cartels has recently been shown to lead to consequences which are counter to our economic intuition. Specifically, it has been shown (see Robert Aumann; Andrew Postlewaite and Robert Rosenthal) that syndication in private goods economies may be disadvantageous to the syndicate members in the sense that all of their imputations in the core of the syndicated economy may be (agent-by-agent) worse or at most no better than any imputation they might receive in the core of the unsyndicated economy. Aumann has even shown examples where all syndicated core points are worse than all unsyndicated core points and in which it is actually disadvantageous to be a monopolist.

In this paper I concentrate on the disadvantages of syndicate formation in public goods economies. I present an example of a simple public goods economy in which syndication is extremely disadvantageous in the sense that the unique core imputation of the syndicated players in the syndicated economy is exactly equal to that Lindahl equilibrium imputation which is most disadvantageous to the syndicate. Put differently, if the players in the syndicate decided not to form a syndicate but rather to act individually and play the "game of perfect competition" (as Jean-Claude Milleron calls it) by truthfully reporting their preferences to an auctioneer who announces parametric prices and allocates cost shares, then their final imputation from such behavior could not be worse than any imputation they would receive by forming a syndicate and bargaining in unison.[1] This result is significant because syndicates, such as labor unions, etc., many times are formed strictly with an eye towards private goods consumption (i.e., salary and fringe benefits). Consequently, even if they are advantageous in that endeavor and do increase the syndicate members' allocation of private goods, their detrimental effects with respect to public goods may cause them, on balance, to be disadvantageous.

In order to analyze this subject intelligently, I will first discuss the difference between what is generally called a "coalition" and what is meant when we use the word "syndicate" in game theory. Then, before presenting the analysis, I will present some problems that exist in the definition of characteristic functions for public goods economies. Finally, an example will be presented which, in a public goods context, exemplifies disadvantageous syndication in the spirit of Aumann, and Postlewaite and Rosenthal. This will be followed by a simple intuitive explanation of exactly why syndication may be disadvantageous in public goods economies.

I. Coalitions and Syndicates

As Lloyd Shapley has pointed out, the concept of blocking in game theory is frequently misunderstood. While the word connotes "foul play" and disruption in its everyday use, in game theory it merely indicates that a set of players who form a coalition to block an imputation are together to get the most for themselves *using only their own resources*. As Lester Telser points out, the unrestricted formation of coalitions is the

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[1]This, of course, implies that all other agents also play the game of perfect competition and truthfully report their preferences so that the true Lindahl equilibrium is reached.

essence of competition and not a sinister or unethical act.

Syndicate formation is quite different, however. In an n-person game, a syndicate is a set of players $S \subset N$ who join and decide to act in unison.[2] Consequently, no subset of S will join a coalition with any member not in S unless all of the players in S also join. If the players who join such a syndicate or union are all of one type (see A. Charnes and Stephen Littlechild; Terje Hansen and Jean Jaskold-Gabzewicz) then the syndicate formed is in essence a monopoly or cartel. We would expect that this would increase (or at least not decrease) their imputation in the associated game over what they would have gotten if they had either not formed a syndicate or acted "competitively." We will find that this need not be true for public goods economies.

II. What a Coalition can Achieve for Itself

In classical game theory the idea of what a coalition can achieve for itself as depicted by the characteristic function is easily defined. Basically, the value of a coalition is that amount of utility R (in the case of transferable utilities) or that set of utility vectors $V_S \subset E^S$ (in the case of nontransferable utilities) that a coalition can guarantee itself *no matter what the remaining players do.* In "orthogonal games," as Shapley and Martin Shubik (1973) call them, this is unambiguously defined. (Market games without externalities are one example of an orthogonal game.) When externalities or public goods are present, the question of what a coalition can achieve for itself is not so easily answered, because the payoff to the coalition is directly related to what the remaining players in the game do. Faced with this problem, Rosenthal pointed out that the classical definition of the characteristic function and consequently of blocking in economies characterized by externalities may not be intuitively appealing. He outlines four types of behavior that can be expected from a countercoalition S^* in a public goods economy (or economy containing externalities) when S forms a coalition. Following Rosenthal and Donald Richter we call these modes of behavior o type, I type, G type, and IG type.

In this paper I will employ the IG and G types of behavioral assumptions only. The G-type behavior has a very simple explanation: if a coalition S forms, it can expect its countercoaliton S^* to take actions which determine group-rational or Pareto optimal imputations for itself. In other words, it is assumed that S^*, having been abandoned by S, will merely do the best it can for itself under the circumstances and maximize its joint utility. Under the IG type of assumption, coalition S^* again organizes its activities in a Pareto optimal or group-rational manner. This time, however, we require that each player's final imputation be individually rational or at least as large as it would be if that player acted individually and accepted the value of the game to himself. Consequently, these assumptions assume a rationality on the part of the countercoalition which states basically that "if you are not with us we will maximize without you even if our group maximization will benefit you. We are not going to hurt ourselves just to hurt you."

There are other types of rationality assumptions that could be made, however; namely the o-type and I-type assumptions. Under the o-type behavior if coalition S forms, it can rely on S^* taking that action which is absolutely worst for S. This may result in a payoff to the members of S^* which is not individually rational, that is, which might reduce their imputation below what they can guarantee themselves by acting alone, but this threat cannot be ruled out. This is the conventional assumption and the one that Duncan Foley used to derive his results. It has led to the nonexistence of the usual limit theorems concerning cores and competitive (in this case Lindahl) equilibria.

Finally, in I-type behavior S^* will counter

[2]The first mention of a syndicate (although the name was not used) was done by John von Neumann and Oskar Morgenstern in a simple three-person trading model, pp. 568-69. More recently, Morgenstern and Gerhard Schwoediauer have reported results which show that the disadvantageousness of syndication will not occur if the von Neumann-Morgenstern solution concept is used as opposed to the core. Also, Michael Maschler has shown that such disadvantageous results will not occur if the bargaining-set solution is used. These findings lead one to question the appropriateness of the core concept.

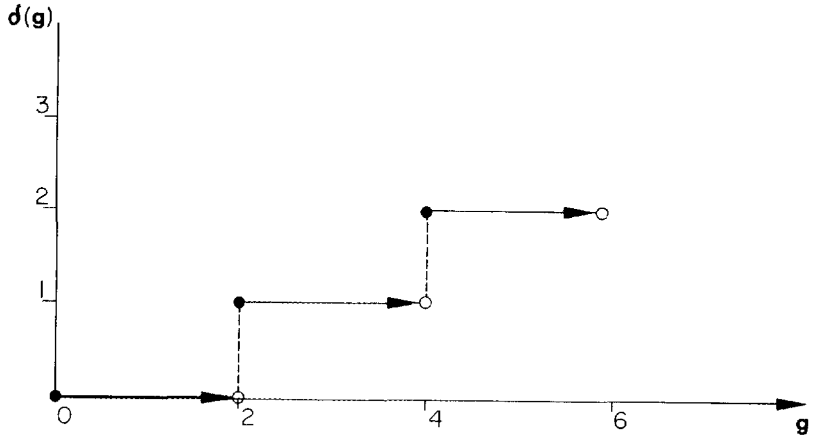

FIGURE 1. PUBLIC GOOD PRODUCTION FUNCTION, $x_2 = \sigma(g)$

by organizing its activities to insure itself at least an individually rational payoff vector, that is, a vector that guarantees each player at least as much as they can guarantee themselves by acting alone.

The reason I do not employ either the o-type or the I-type rationality assumptions in my analysis is simple. The o-type behavior is purely spiteful behavior. Consequently it may require the countercoalition to "cut off its nose to spite its face" and such behavior may have disastrous consequences for coalition S^*. In addition, such an assumption was shown by Rosenthal to lead to unsatisfactory results. The I-type behavior, on the other hand, calls for a partial rationality which I feel is less desirable than the rationality described by either the G or the IG types of assumptions. In any case, whatever type of assumption the reader feels is preferable, this analysis will only concentrate on the G or IG type of assumption.

III

Using our rationality concepts, let us look at a simple example of a public goods economy to get an intuitive idea of why syndicate formation may be disadvantageous.

Let E be an economy with a set of three identical traders N indexed $i = 1, 2, 3$, all characterized by the following utility function:

$$U^i = (x_1^i)^{1/z} + bx_2 \tag{1}$$

where[3] $z > 1$, $b > 0$; x_1^i is the amount of private good x_1 that the ith individual consumes and x_2 is the total amount of a pure public good produced in the economy. Assume an initial endowment as follows:

$$w^1 = (x_1^1, g) = (1, 1) \tag{2}$$

$$w^2 = (x_1^2, g) = (1, 1) \tag{3}$$

$$w^3 = (x_1^3, g) = (1, 1) \tag{4}$$

where x_1^i is the private good in i's possession and g an all-purpose good[4] yielding no utility but which can be transformed into either the private good x_1 or the public good x_2 by the following production functions:

$$x_1 = \Psi(g) = (1/\gamma)(g) \quad \gamma > 0 \tag{5}$$

$$x_2 = \delta(g) = K/2 \tag{6}$$

where K is the closest even integer not greater than g. The function $\delta(g)$ states that the production function for x_2 is a step function with steps at all even integers. This is depicted in Figure 1.

From the model's description, it is clear that x_1 will never be traded and that the only reason to form coalitions in this economy would be to produce the public goods x_2. Let us assume that the parameters of the model

[3]The fact that the public good appears in each individual's utility function as a linear additive term is merely a convenience and not necessary for the example to work.

[4]The term g may be considered an endowment of labor in an economy where leisure has no utility.

(b, γ, and z) are such that the best a coalition of two can do for itself is to take its two units of g and transform them into one unit of x_2 via δ. This is equivalent to assuming that the sum of the marginal rate of substitution of x_1 for x_2 for the two players evaluated at $x_1^i = 1$, $x_2 = 0$ is strictly less than the marginal rate of transformation. Using the IG or G rationality assumption defined above we can try to specify a characteristic function that would describe this economy. However, such an attempt would fail since the function would not be superadditive.

To demonstrate why this is so, consider the following description of the utility vector or vectors that each coalition in our economy can achieve for itself assuming either our IG or G assumption, and in addition assuming that no transfers of the existing endowments of private goods (the x_1^i's) are allowed between agents:[5] First each individual agent can guarantee himself $(1 + 1/\gamma)^{1/z} + b$ since he can assume that the counter coalition (a coalition of two agents) is capable of producing one unit of the public good (since they have two units of g between them), and producing one unit is the group rational thing for them to do, by assumption. A coalition of two, say i and j, cannot rely on the countercoalition producing any of the public good. Even if the counter-coalition wished to, it only has one unit of g. Consequently, the singleton countercoalition would produce no public good, and transform all of its g into $(1/\gamma)$ units of the private good. The coalition of two could then guarantee itself only the following single utility vector, $x = (1 + b, 1 + b)$, in which each contributes one unit of g to build the public good and is left with one unit of the private good and one unit of the public good to consume. Finally, the grand coalition faces the null set as a countercoalition. Since it obviously can not rely on it to provide any of the public good, it will have to produce it itself through contributions from the members of the economy. To finance the public good they would have to collect two units of g from the three of them. Consequently, the set Y of utility vectors of the form

[5]This is, of course, just a simplifying assumption.

$$(7)\quad Y = \Big\{y \,|\, y = \Big[\Big(1 + \frac{1-a_1}{\gamma}\Big)^{1/z} + b,\ \Big(1 + \frac{1-a_2}{\gamma}\Big)^{1/z} + b,\ \Big(1 + \frac{1-a_3}{\gamma}\Big)^{1/z} + b\Big]\Big\}$$

$$0 \le a_i \le 1,\ \sum_{i=1}^{3} a_i = 2$$

constitute the set of utility vectors achievable by the grand coalition. Written out formally, what has just been described appears as

$$(8)\quad V((i)) = (1 + 1/\gamma)^{1/z} + b \quad i = 1, 2, 3$$

$$(9)\quad V(i,j) = x = (1 + b, 1 + b)$$

$$(10)\quad V(123) = Y = \Big\{y \,|\, y = \Big[\Big(1 + \frac{1-a_1}{\gamma}\Big)^{1/z} + b\Big),\ \Big(1 + \frac{1-a_2}{\gamma}\Big)^{1/z} + b\Big),\ \Big(1 + \frac{1-a_3}{\gamma}\Big)^{1/z} + b\Big),\Big]\Big\}$$

$$0 \le a_i \le 1,\ \sum_{i=1}^{3} a_i = 2$$

It is easy to see that this is not a characteristic function since, for example, $V(i \cup j) < V(i) + V(j)$. However, this does not present a problem since we are interested only in the Lindahl equilibrium of this economy which is not a game-theoretical concept and can be defined independently of the characteristic function. The imputations associated with it appear as

$$(11)\quad L = \Big\{l \,|\, l = \Big[\Big(1 + \frac{1-a_1}{\gamma}\Big)^{1/z} + b\Big),\ \Big(\Big(1 + \frac{1-a_2}{\gamma}\Big)^{1/z} + b\Big),\ \Big(\Big(1 + \frac{1-a_3}{\gamma}\Big)^{1/z} + b\Big]\Big\}$$

$$0 \le a_i \le 1,\ \sum_{i=1}^{3} a_i = 2$$

This set L of Lindahl imputations, has a very simple explanation. Each $l \in L$ is characterized by a different vector of contributions

(a_1, a_2, a_3) where a_i specifies the amount of good g player i is being asked to contribute towards the construction of the public good. Since the sum of these contributions is 2, exactly one unit of the public good will be constructed. The remaining units of g that each agent has after paying this requested contribution, namely $(1 - a_i)$, can be used to produce $(1 - a_i)/\gamma$ units of x_1, the private good, through the production function $\Psi(g)$. Any vector (a_1, a_2, a_3) such that $0 \le a_i < 1$ and $\Sigma_{i=1}^3 a_i = 2$, determines an imputation in L because, at the individualized prices defined by any such vector, each agent would maximize his utility by contributing his called-for share and accepting the resulting bundle of public and private goods. In other words, at the announced vector of contributions each agent in the economy would prefer to have the bundle of private and public goods defined by the associated vector in L than any other bundle he could afford to consume at those implicitly defined prices. The resulting allocations are Lindahl allocations.[6]

To investigate the effects of syndication on the economy, let agents 1 and 2 form a syndicate and call them *Syn*. If we now tried to specify a characteristic function for the game defined by this economy, we would find that it would indeed exist, although it would define an inessential game or a game in which there was no incentive for coalition formation.

[6]It is interesting to note that imputations associated with the Lindahl equilibrium are identical to the core of the game defined by the characteristic function which assumes o-type rationality. Its characteristic function appears as

$$V((i)) = \left(1 + \frac{1}{\gamma}\right)^{1/z}$$

$$V(ij) = x = (1 + b, 1 + b)$$

$$V(123) = Y = \left\{y \,|\, y = \left(\left(1 + \frac{1 - a_1}{\gamma}\right)^{1/z} + b\right), \left(1 + \frac{1 - a_2}{\gamma}\right)^{1/z} + b\right), \left(1 + \frac{1 - a_3}{\gamma}\right)^{1/z} + b\right)\right\}$$

$$0 \le a_i \le 1, \sum_{i=1}^{3} a_i = 2$$

The reason why a characteristic function exists here and has a nonempty core is because of the o-type assumption which makes larger coalitions beneficial.

To see this, again assume the *IG* or *G* assumptions and make the usual assumption that syndicate members are treated equally within the syndicate (i.e., they share equally their joint contribution of g to the construction of the public good). We can define the characteristic function for this game as follows:

$$V(3) = \left(1 + \frac{1}{\gamma}\right)^{1/z} + b \tag{12}$$

Thus player 3 can rely on *Syn* to build one unit of the public good and it uses all of its g to build x_1.

$$V(Syn) = x = (1 + b, 1 + b) \tag{13}$$

Thus the syndicate can not rely on any public good construction by the singleton player 3 and maximizes its joint utility by building one unit of the public good.

$$V(3, Syn) = X = \tag{14}$$

$$\left\{x \,|\, x = \left[\left(1 + \frac{1 - a_3}{\gamma}\right)^{1/z} + b, \left(1 + \frac{1 - \frac{2 - a_3}{2}}{\gamma}\right)^{1/z} + b, \left(1 + \frac{1 - \frac{2 - a_3}{2}}{\gamma}\right)^{1/z} + b\right]\right\}$$

$$0 \le a_3 \le 1$$

Here, the grand coalition can achieve any utility vector $x \in X$ that is determined by joint contributions of g which sum to 2 in which the syndicate splits the contribution not made by player 3.

Since this game is inessential, the core is nonempty, is unique, and is represented by the vector,

$$x = \left(1 + b, 1 + b, \left(1 + \frac{1}{\gamma}\right)^{1/z} + b\right) \tag{15}$$

which occurs when the syndicate alone builds the public good and player 3 gets a total "free ride." This is obviously the only core imputation. Any imputation that required player 3 to contribute a positive amount could be blocked by that player acting alone. This unique core imputation, however, is at the extreme end of the set of unsyndicated Lindahl equilibria when viewed from the point of view of the syndicate's members. In other words, syndication determines a core imputation in which $a_1 = a_2 = 1$, $a_3 = 0$, or one in which the syndicate finances the entire public good by themselves without any contribution from the unsyndicated player. This is worse than they would have done if they had not formed a syndicate and accepted the imputation associated with any announced Lindahl equilibria. Any other unsyndicated Lindahl equilibria is at least as good for them.

IV. Why Syndication may be Disadvantageous

The explanation of why syndicates do so poorly in my example is quite simple. Basically it is because syndicates create an indivisibility in the bargaining process that works to the detriment of the syndicate's members. The reason for this is as follows:

In our simple economy, under any of our rationality assumptions *G* or *IG*, there exists an "optimal" size of coalition which maximized the blocking power of the players in it by maximizing the free ride they receive from the rest of the economy. In my example this is a coalition of size one. Before syndication, each player had an equal opportunity to form such a coalition or at least had an equal threat to. However, once the syndicate actually formed, the members in it lost a very powerful threat since none of them could ever be part of a coalition of this size. They had transformed themselves into an indivisible player whose size prevented them from using a bargaining threat that each one had separately before syndication. At the very outset of bargaining they are "too big."

The fact that the members of the syndicate have lost some of their bargaining or blocking power should be obvious simply from the fact that the syndicated game has a nonempty core, while the unsyndicated game does not. This can be attributed strictly to the fact that the syndicated players are less able to block imputations in the syndicated game than they are in the unsyndicated game. This is so because neither of the syndicated players can form a singleton coalition after syndication, and given our rationality assumptions, singleton coalitions have the maximum blocking ability since they have threats that large syndicates do not have.

V. Conclusion

The conclusion of this paper is simple: In economies that contain both public and private goods, syndication is a two-edged sword. While it might be beneficial to its members in providing them with a greater amount of private goods (salary, fringe benefits, retirement programs), it may diminish their utility in the possession of public goods since their size prevents them from getting the free ride that smaller agents can achieve. In forming a syndicate then, these various costs and benefits must be weighed.

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