Nominal bargaining power, selection protocol, and discounting in legislative bargaining

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Abstract

The comparative static predictions of the Baron and Ferejohn [Baron, D.P., and Ferejohn, J.A., (1989). Bargaining in legislatures, American Political Science Review 83 (4), 1181–1206] model better organize behavior in legislative bargaining experiments than Gamson’s Law. Regressions similar to those employed in field data produce results seemingly in support of Gamson’s Law (even when using data generated by simulating agents who behave according to the Baron–Ferejohn model), but this is determined by the selection protocol which recognizes voting blocks in proportion to the number of votes controlled. Proposer power is not nearly as strong as predicted in the closed rule Baron and Ferejohn model, as coalition partners refuse to take the small shares given by the continuation value of the game. Discounting pushes behavior in the direction predicted by Baron and Ferejohn but has a much smaller effect than predicted.

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1. Introduction

The legislative bargaining process is central to the allocation of public resources. A complete characterization of the bargaining process is bound to be quite complicated, so
that any model must abstract from some features of reality. Nonetheless, modeling is central to our understanding of the bargaining process as it allows us to focus on the central forces at work and to determine the effects on bargaining outcomes of key variables such as the impatience of legislators, the voting rules employed, and the impact of unequal bargaining power between voting blocks.

The present paper looks at the legislative bargaining process focusing on the effect of changes in nominal bargaining power, the selection protocol, and the discount rate on bargaining outcomes. The theoretical framework for the experiment is the Baron and Ferejohn (1989) model (hereafter BF model) which is the most frequently used formal model of legislative bargaining. The BF model has been applied to a number of situations ranging from special interest politics (Bennedsen and Feldmann, 2002; Persson, 1998) to social choice issues (Banks and Duggan, 2000). Some of these applications deal with central issues in public economics. For instance, Baron (1991) extends the model to show how it can explain the existence of socially inefficient programs. Alesina and Perotti (1996) use insights from the model to explain the presence or absence of fiscal discipline in parliamentary democracies.

The BF model predicts substantial proposer power, unaffected by “nominal changes” in voting weights, i.e., changes that do not affect real bargaining power. In other words, according to the predictions of the BF model, the prime minister’s party (the proposer) should have a disproportionate share of cabinet positions, and this share does not depend on the relative voting weights, as long as the equivalent homogeneous representation of the game is unchanged. This property of “insensitivity” to nominal changes in voting weights is common to all the noncooperative bargaining models in the literature, and contrasts with the empirical studies of coalition governments, where it has been argued that the data is closer to a proportional relationship between the ministerial payoffs for coalition members and the nominal votes each coalition partner contributes to the coalition (see Warwick and Druckman, 2001).¹ This proportional relationship between relative votes and bargaining outcomes was first suggested by Gamson (1961a), and is commonly referred to as Gamson’s Law (hereafter GL). It is not based on any explicit game theoretic formulation of the legislative bargaining process, but rather owes its importance to the strong empirical regularity reported between cabinet posts and votes contributed to the ruling coalition in parliamentary democracies.²

The present experiment compares the predictions of the BF model with GL in a divide the dollar game with three legislative voting blocks. With three voting blocks, none of which has a majority by itself, each voting block (regardless of the number of votes it controls) has equal real bargaining power within any majoritarian bargaining model: In fact, the approval of an allocation always requires a coalition of two out of three voting blocks. Hence, in the BF model, changes in the relative numbers of votes that do not result

¹ Warwick and Druckman (2001) improve on the methodology of Browne and Franklin (1973) by controlling for the importance or saliency of the portfolios each party receives. They too, however, conclude that the relationship is more or less proportional.

² Morelli’s (1999) demand bargaining model predicts proportionality between legislative bargaining outcomes and real (not nominal) bargaining power. This is not related to GL because the latter stresses the prevalence of proportionality with respect to the nominal weights.
in any party achieving an outright majority have no effect on the ex post distribution of benefits among coalition partners.\(^3\) In contrast, according to GL, there is no distinction between real and nominal bargaining power. This is clear from Gamson’s own writings as well as the empirical analysis of coalition governments supporting GL.\(^4\)

Our results identify statistically significant proposer power under all treatments, although the magnitude of proposer power is substantially smaller than predicted under the BF model. Moving from equal to unequal nominal bargaining power has no significant effect on the ex post distribution of benefits among coalition partners, consistent with the predictions of the BF model, against GL. Moreover, changes in the probability with which voting blocks’ proposals will be recognized does alter coalition composition in the way the BF model predicts, as opposed to the no change prediction of GL.

Employing regressions mimicking those applied to field data, we find clear evidence supporting GL, although the comparative static outcomes of the experiment clearly favor the BF model. This seemingly inconsistent results are reconciled in this paper: the typical regression specification employed in field data analysis do not properly account for the formateur rules typically observed in government coalition formation procedures (see Diermeier and Merlo, 2004).

There have been only limited experimental investigations of the BF model prior to this. McKelvey (1991) investigated the closed rule BF model with three voters choosing between three or four predetermined allocations (resulting in a mixed strategy equilibrium), and with a discount rate of 0.95. Deviations from predicted behavior included a reluctance to propose to coalition partners alternatives that would put them close to their continuation payoff.\(^5\) In contrast, our experimental design implements an infinite horizon game with and without discounting, which always yields a pure strategy equilibrium with respect to coalition members’ shares. (Our treatments yield mixed strategy equilibria with respect to coalition composition.) Diermeier and Morton (2004) investigate the BF model focusing on varying recognition probabilities and on the share of votes that each elector controls under closed rule procedures. Each election consists of a finite number of rounds (5), with a zero payoff if no agreement was reached in the last round. They report that coalition member shares are more equal than predicted under Baron-Ferejohn and that in one treatment they are proportional to the votes they control. In contrast, our comparative static outcomes clearly favor the BF model over proportionality with respect to nominal voting weights.

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\(^3\) In coalitional bargaining games with more than three players, a change in real bargaining power occurs when a change in relative weights alters the number of minimal winning coalitions each player can belong to (see Morelli, 1999; Schofield and Laver, 1985).

\(^4\) See Gamson (1961a, p. 567) “Convention” 2, and Browne and Franklin (1973, p. 457). Browne and Franklin, as well as Warwick and Druckman, make no distinction between real and nominal bargaining power. Schofield and Laver (1985) and Laver and Schofield (1990) find that a measure of real bargaining power based on cooperative solutions of the family of the bargaining set performs better than GL in some cases. Morelli (1999) and Morelli and Montero (2003) provide a rational foundation for a proportionality norm based directly on the real bargaining weights. We contrast the predictions of the BF model against these models of “real” proportionality in Fréchette et al. (2003b).

\(^5\) Bolton et al. (2003) find that in three-person super additive coalition games, bargainers with a monopoly on communication (so that they are effectively proposers) do not always obtain as large a share of the pie as game theory predicts.
Fréchette et al. (2003a; hereafter FKL) is closest in design to the present experiment. They focus on the impact of closed versus open amendment rules on legislative outcomes. There are a number of differences between the present experiment and FKL. Here we employ only closed rule procedures and vary voting block size, whereas FKL always had equal-sized voting blocks. Failure to achieve an allocation in the first round of voting in FKL resulted in a shrinking pie (discounting) whereas the main treatments here employ no discounting. This last difference is potentially quite important because the key strategic factor distinguishing the BF model from the bilateral bargaining games of Rubinstein (1982) and Binmore (1986) is that multilateral bargaining does not require a shrinking pie to generate an equilibrium. Rather, the key driving force is the exclusion of some voters from the winning coalition. This is predicted to motivate coalition members to vote for proposals although they are getting a substantially smaller share of the pie, as otherwise they risk being excluded from the coalition in the next round of proposals (although they also have a chance to be the proposer and to exercise proposer power). In keeping the size of the pie constant, we isolate the impact of this strategic factor. We then replicate our baseline treatment with a shrinking pie to determine the effects of discounting.

The paper proceeds as follows. The next section develops the BF model for our experimental design, and derives the stationary subgame perfect Nash equilibrium for any selection protocol. Our experimental procedures and treatment effects are reported next, followed by the experimental results. Section 7 recaps our main findings.

2. The Bargaining Model

We consider a three-party game where any majority coalition—at least two out of three—can decide how they should share a dollar. Each party has a potentially different nominal weight, \( w_i < (n/2) \forall i \), with \( \sum_{i=1}^{3} w_i = n \), where \( n \geq 3 \) is the total number of seats in the assembly and \( w_i \) is the number of seats held by party \( i \). Think of the dollar to be divided as the total amount of ministerial payoffs available to a coalitional government.

For any configuration of these weights, the three players always maintain equal real bargaining power because \( w_i < (n/2) \forall i \) guarantees that no party can determine the payoff sharing without agreement with another party.\(^6\) Thus, the distribution of nominal weights does not affect the distribution of equilibrium payoffs.

A complete theoretical treatment of weighted majority games is beyond the scope of this paper. We just mention that this type of game has been at the center of cooperative game theory since von Neumann and Morgenstern (1944), and became the focus of noncooperative bargaining theory in the late 1980s. The BF model is the noncooperative bargaining model most used in the political science and public economics literature, and is the only game theoretic model considered in this paper.

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\(^6\) Using the terminology of cooperative game theory, the minimum integer homogeneous representation of this game is 1,1,1.
Let the three parties be the relevant three players of the bargaining game. One player is randomly selected to be the proposer; (s)he makes a proposal to another player on how to share the dollar, and, if the offer is accepted, the game is over. If the offer is rejected, then another random selection of a proposer is made, and so on.\footnote{Here we provide the explicit theoretical prediction only for the no-discounting case, and discuss what changes with discounting at the end of the section.}

Looking at the infinite horizon version of such an alternating-offer bargaining model, we focus on the same solution concept as BF, Stationary Subgame Perfect Equilibria (SSPE).

A key variable is the so-called protocol; that is, the probabilities with which the players are selected to be the next proposer when a proposal is rejected. Consider any protocol \( \rho = \rho_1, \rho_2, \rho_3 \), with \( \rho_i > 0 \) \( \forall i \) and \( \sum \rho_i = 1 \). We will derive the prediction of the theory for every \( \rho \), but the experiments will focus on two focal protocols, the egalitarian one, \( \rho_i = (1/3)^i \forall i \), and the proportional one, \( \rho_i = (w_i/n) \). The first one is important because, in this case, with \( w_i < (n/2) \) for all \( i \), the bargaining power in the absence of institutions is equal, regardless of the weights. The second protocol is important because it fits the institutional norm (Diermeier and Merlo, 2004). Although the equilibrium is derived only for the special case of three voting blocks, it is noteworthy that this is the first paper to characterize the SSPE for any selection protocol.\footnote{A partial characterization can be found in the section of BF entitled “An Application: Government Formation in Parliamentary Systems”.}

Given that the three players have equal real bargaining power, it is natural to allow a proposer to mix on whom to propose. A stationary strategy for player \( i \) can be summarized by (1) the offer \( \chi_i^j \in [0,1] \) (s)he would make to player \( j \) at every node where (s)he is the proposer, and (2) the probability \( p_i^j \) that \( i \) makes the offer to \( j \). For a responder (i.e., for a player who has been made an offer and is called to respond), the only payoff-relevant information is the offer received. Hence, the stationary strategy of any player \( i \) includes the tuple \( \chi_i^j, \chi_i^k, p_i^j \) (for when \( i \) is the proposer) and an acceptance threshold \( \chi' \), below which offers are rejected. We will use the term ex ante equilibrium payoffs to indicate the expected payoffs associated with an equilibrium strategy profile before the identity of the first proposer is revealed.

**Proposition 1.** Consider the three-player infinite horizon closed rule bargaining game without discounting described above.

For every interior protocol \( \rho \) and for every distribution of weights \( w \):

(I) Ex ante payoffs: All the SSPE of the game determine a unique egalitarian distribution of ex ante payoffs, coinciding with the Nucleolus of the game.

(II) Equilibrium offers: In every SSPE, any player \( i \) recognized to make a proposal, offers \( \chi_i^j = 1/3 \) to a chosen responder \( j \), and \( \chi_i^k = 0 \), \( k \neq j \), and is indifferent between the two other players when choosing the responder \( (j) \); the offer is accepted, and hence the payoff for the proposer is \( (2/3) \). Moreover, in every SSPE profile, the acceptance threshold for every player \( i \) is \( \chi' = (1/3) \).
(III) Equilibrium probabilities with which responders are chosen: A triplet \((p_k^j, p_j^i, p_i^k)\) of mixing probabilities suffices (the other three are implicitly derived). Every SSPE is characterized by one such triplet, and the set of triplets corresponding to SSPE is identified by the following system:

\[
\begin{align*}
    p_k^j &\in \left[ \max \left( 0, \frac{1 - \rho_i - 2 \rho_j}{1 - \rho_i - \rho_j}, \frac{\rho_i - \rho_j}{1 - \rho_i - \rho_j} \right), \min \left( 1, \frac{\rho_i}{1 - \rho_i - \rho_j}, \frac{1 - 2 \rho_j}{1 - \rho_i - \rho_j} \right) \right], \\
    p_j^k &= \frac{\rho_i - p_k^j + p_k^j \rho_i + p_k^j \rho_j}{\rho_j}, \\
    p_i^k &= -\frac{1 - \rho_i - 2 \rho_j - p_j^k \rho_i + p_j^k \rho_j}{\rho_i} \\
\end{align*}
\]

**Proof.** See Appendix. □

In every SSPE, regardless of the protocol and the nominal weights, the proposer offers \((1/3)\) to someone, and keeps \((2/3)\). However, the set of mixing probabilities with which proposers choose responders in SSPE depends on the protocol. With the egalitarian protocol, the range of such mixtures is identified by

\[
\begin{align*}
    p_k^j &= p_i^k \in [0,1] \\
    p_j^k &= (1 - p_k^j) \\
\end{align*}
\]

With a proportional protocol, with the specific weights \(w_i = 45, w_j = 45, w_k = 9\) employed in our experimental design, the system identifying the mixing probabilities is

\[
\begin{align*}
    p_k^j &\in [0,1] \\
    p_j^k &= 1 - \frac{1}{5} p_k^j \\
    p_i^k &= \frac{4}{5} + \frac{1}{5} p_k^j \\
\end{align*}
\]

Because \(w_i = w_j\), a natural benchmark is the unique symmetric SSPE profile of such probabilities, which is obtained by imposing \(p_j^k = p_i^k\), with the point prediction \(p_k^j = (1/2), p_j^k = p_i^k = 0.9\).

Finally, we should mention that the only difference (with respect to Proposition 1) when discounting is introduced is in terms of the proposer’s power: for
example, with a discount factor $\delta=0.5$, the proposer’s ex-post payoff is $(5/6)$ rather than $(2/3)$.

3. Experimental design

In each election, three subjects divided $30 between three voting blocks, with one subject representing each voting block. Election procedures were as follows: First, all subjects entered a proposal allocating the $30. The one proposal was randomly selected to be the standing proposal. This proposal was posted on subjects’ screens giving the amounts allocated to each voting block, by subject number, along with the number of votes controlled by that subject. Proposals were voted up or down, with no opportunity for amendment. If a simple majority accepted the proposal, the payoff was implemented and the election ended. If the proposal was rejected, the process repeated itself (after applying the discount rate, if there was one).

Experimental treatments are reported in Table 1. In each election, there were a total of 99 votes divided between the three voting blocks, with all the votes within a block being cast as a block. In the baseline treatment (equal weights and equal selection probabilities, henceforth EWES), each voter controlled 33 votes and had a 1/3 chance of their proposal being selected to be voted on. The next two treatments both involved two subjects, each controlling 45 votes and one subject controlling 9 votes.

In the UWES treatment (unequal weights and equal selection probabilities), each voting block continued to have a 1/3 probability of their proposal being recognized and voted on. The next two treatments both involved two subjects, each controlling 45 votes and one subject controlling 9 votes.

In the UWUS treatment (unequal weights and unequal selection probabilities), each voting block continued to have a 1/3 probability of their proposal being recognized and voted on. Within the framework of the BF model, this treatment tests if there are any framing effects, or other unanticipated effects, resulting from perceived differences in bargaining power, as there is no change in real bargaining power. Furthermore, because recognition probabilities

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9 For any discount rate $\delta$, the continuation payoff of any player, if a proposal is rejected and a new proposer has to be recognized, is equal to $\delta$ times the ex ante payoff, which is always $(1/3)$. Hence, the proposer can retain in equilibrium $1-\delta(1/3)$.

10 In both cases, subjects’ weights, which were selected randomly during the dry run, remained fixed throughout the experimental session.
are the same as in the EWES treatment, BF predicts that the composition of minimal winning coalitions (MWCs) will be independent of voting block size. In contrast, GL predicts a dramatic effect on the distribution of payoffs within any MWC as well as on the composition of that coalition: Shares in any MWC should be proportional to the number of votes contributed to the coalition, resulting in shares of $1/2$, $1/2$ in coalitions comprised of the two 45-vote blocks, and shares of $(45/54)$, $(9/54)$ in coalitions consisting of one 45-vote block and the 9-vote block. Furthermore, all MWCs should include the 9-vote block, as the 45-vote block receives a larger share of the benefits when partnering with the 9-vote block ($(45/54)$ vs. $(1/2)$).  

In the UWUS treatment (unequal weight and unequal selection probabilities), the protocol is proportional to the number of votes each block controls. Here too BF predicts no differences in either ex-ante or ex-post shares of the different voting blocks compared to the EWES treatment. However, BF does predict that both 45-vote blocks will have a strong preference for including the 9-vote block in their proposals (anywhere between 80% and 100% of the time). Furthermore, if we assume symmetry between the two 45 voting blocks, then the point prediction for partnering with the 9-vote block is 90%. In contrast, GL predicts no impact from the UWUS treatment compared to the UWES treatment.

The last treatment replicates the EWES treatment but with a discount rate of 0.5. Within the BF model, proposals should continue to be passed in the first round of each election, but the discounting increases the proposer’s power so that the ex-post distribution of benefits under the SSPE is $((5/6)$, $(1/6)$, 0) (with share to the proposer listed first). Predictions under GL are unaffected by the discount rate.

To minimize the possibility of repeated play effects, we recruited between 12 and 18 subjects per session, conducting between 4 and 6 elections simultaneously. Subjects were assigned to each “legislative” cohort randomly in each election, subject to the restriction that in elections with unequal voting blocks each cohort contained two 45-vote blocks and one 9-vote block. Subject numbers also changed randomly between elections (but not between rounds of a given election). Feedback from voting outcomes was limited to a subject’s legislative cohort. This feedback consisted of the proposed distribution of benefits in each round of an election, which voted for or against the distribution, and whether the distribution passed or failed along with the vote totals.  

Subjects were recruited through announcements in undergraduate classes, advertisements in student newspapers, and e-mail announcements at the Ohio State University. For each treatment, there were 2 inexperienced subject sessions and 1 experienced subject session. A total of 11 elections were held in each inexperienced subject session, 1 dry run, and 10 elections for cash, with one of the cash elections selected at random to be paid off on. Subject payments from this one election were equal to the money allocated to their

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11 This is made clear in the following passage: “where the total payoff is held constant, he [a player choosing a coalition] will favor the cheapest winning coalition.” Gamson (1961b, p. 376).

12 Screens also displayed the proposed shares and votes for the last three elections as well as the proposed shares and votes for up to the past three rounds of the current election. Other general information, such as the discount rate, the number of votes required for a proposal to be accepted, etc., was also displayed. Screen shots, along with instructions, are provided at the website [http://www.econ.ohio-state.edu/kagel/bf3instructions.pdf](http://www.econ.ohio-state.edu/kagel/bf3instructions.pdf).
voting block in that election.\textsuperscript{13} In addition, each subject received a participation fee of $8. Subjects were told that sessions would last approximately 1.5–2.0 h. None of the sessions required intervention by the experimenters to end within this timeframe, with most sessions ending within 1.5 h, including time for the instructions and the dry run.

4. Results for $\delta=1$ treatments

We report results in terms of a series of conclusions, each followed by the supporting data. We begin with conclusions that apply to all three treatments.

\textbf{Conclusion 1.} A majority of proposals are accepted without delay, as the BF model predicts. However, delays persist until the end.

Table 2 reports the percentage of proposals that were accepted in round 1. It gives the results for all elections and for the last three elections.\textsuperscript{14} These percentages are relatively high, averaging some 68\% (77\%) for the three treatments combined for inexperienced (experienced) subjects. Averaging over all treatments, the average number of rounds goes from 1.6 for inexperienced voters to 1.3 for experienced, with the number of rounds rarely exceeding 2 for experienced voters.

\textbf{Conclusion 2.} A majority of proposals are for minimal winning coalitions.

On average, 69\% of the proposals for inexperienced voters are for MWCs, with this number increasing to 85\% for experienced voters. Table 3 breaks these numbers out by treatment. Very few offers are perfectly egalitarian, only 7\% for inexperienced subjects and 5\% for experienced subjects.

\textbf{Conclusion 3.} Proposers receive a uniformly larger share of the benefits than coalition members, so that we can reject a null hypothesis of no proposer power. Nevertheless, proposers take well below the SSPE prediction in all three treatments and well below the prediction for GL for the UWES and UWUS treatments.

The average share of the proposer for accepted offers is given in Table 4.\textsuperscript{15} Inexperienced voters in their role as proposers obtain an average share of .51 for

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|}
\hline
Elections & EWES & UWES & UWUS \\
\hline
Inexperienced & All & 65 & 73 & 65 \\
 & Last 3 & 67 & 80 & 67 \\
Experienced & All & 77 & 85 & 68 \\
 & Last 3 & 78 & 78 & 67 \\
\hline
\end{tabular}
\caption{Percentage (%) of elections ending in round 1}
\end{table}

\textsuperscript{13} The dry run was eliminated in the experienced subject sessions.

\textsuperscript{14} Unless stated otherwise, we report data for all proposals, whether they were selected to be voted on or were actually passed.

\textsuperscript{15} Conditioning on accepted offers that are MWCs, proposer’s shares are only slightly more: 0.55 at both experience levels in EWES, 0.54 and 0.52 in UWES for inexperienced and experienced voters, respectively, and 0.56 and 0.55 in UWUS.
themselves, compared to the next highest average share of 0.43. For experienced voters, these numbers are 0.52 vs. 0.45. (These numbers add to less than 1 because of super majorities.) For all treatments and experience levels, using a sign test (Snedecor and Cochran, 1980), the null hypothesis that the median of the differences between the proposer’s share and the share offered to anyone else is zero can be rejected at the 5% level.\footnote{These tests are performed using subject averages. Unless otherwise specified, all tests reported use subject averages, thereby mitigating “repeated measure” problems.}

The data in Table 4 show that proposers shares are quite far away, on average, from the 2/3 predicted under the SSPE, with relatively few SSPE proposals overall-12% (11%) for inexperienced (experienced) voters, of which about half (a third) were accepted for inexperienced (experienced) voters.\footnote{Among inexperienced subjects, there are a number of proposals giving more than two-thirds to the proposer, but these were essentially eliminated for experienced subjects. One subject consistently proposed giving all the money to one player, sometimes himself and sometimes to others. This outlier has been dropped from the analysis throughout. The percentage of SSPE offers accepted is computed over the offers that took the floor.} Although proposer’s shares are close to the prediction for GL for the EWES treatment, they are far from the average predicted under the UWES and UWUS treatments (62% and 78%, respectively).\footnote{These percentages account for the probability of being selected as the proposer in conjunction with shares predicted for each block.} Average shares of accepted offers are approximately constant across treatments, so that the comparative static predictions of the BF model are satisfied while those of GL are not.\footnote{This is true when averaging over all proposed shares as well.}

Average shares of accepted offers are approximately constant across treatments, so that the comparative static predictions of the BF model are satisfied while those of GL are not.\footnote{This is true when averaging over all proposed shares as well.}

Conclusion 4. Voting for or against a proposal is almost exclusively based on own share of the benefits, with minimal concern for the shares of the least well off or for the proposer’s share. Shares below 1/3 are almost always rejected and shares above 1/3 are usually accepted. However, in a large number of cases, shares between 1/3 and 7/15 (between $10 and $14) are rejected.

Fig. 1 pools the data between the three treatments, but distinguishes between inexperienced and experienced subjects. Offers below 1/3 ($10) are rejected 97% (98%)
of the time for inexperienced (experienced) subjects. Note that GL predicts that shares of $5 or more will be accepted under the UWES and UWUS treatments by the 9-vote block. However, this is clearly not the case. Furthermore, although the BF model predicts that offers of $1/3 or more will be accepted, and they are a majority of the time, offers between $10–$14.49 are rejected 26% (6%) of the time for inexperienced (experienced) subjects.20

Table 5 reports estimates of the following voting equation:

\[
\text{vote}_{it} = I\{\beta_0 + \beta_1 x_{it} + \beta_2 PS_{it} + \beta_3 SZ_{it} + \alpha_i + m_{it} z_0 \geq 0\}
\]  

(5)

where \(I\{\cdot\}\) is an indicator function that takes value 1, if the left-hand side of the inequality inside the brackets is greater than or equal to 0, and 0 otherwise. Explanatory variables include own share (\(s_{it}\)), SZ (the single-zero strategy), an indicator variable taking value one if at least one subject is totally excluded from the division of the benefits in the proposal on the floor, and the share the proposer takes (PS). The equation is estimated using a random-effects probit, with a one-way subject error component for all rounds.21 The sign of the coefficient for own share is positive, large in value relative to the other coefficients, and statistically significant, except for the EWES treatment with experienced subjects where nothing is statistically significant.22 The coefficients for the SZ strategy, and for proposers share (PS), are not statistically significant except for the EWES inexperienced voter treatment where PS is significant at the 10% level. The implication is that subjects are primarily voting out of concern for their own share of the benefits, with little or no concern for the shares of the least well off and for the proposer’s share. As for differences in vote patterns for 9-vote vs. 45-vote blocks, using likelihood ratio tests, the null hypothesis that voting is independent of block size cannot be rejected except for the UWUS treatment within experienced voters. These and other differences between the 9- and 45-vote blocks will be explored in more detail below.

**Conclusion 5.** There are minor differences in behavior between EWES and UWES treatments for inexperienced voters. These differences are, however, no longer present for experienced voters. These comparative static results support the BF model over GL.

Recall that, because real bargaining power and the selection protocol are the same between the EWES and UWES treatments, the BF model predicts no difference in

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20 There are very few proposals between $14.50 and $14.99 of which 7% (4%) were rejected for inexperienced (experienced) voters.

21 The null hypothesis of no random-effects can be rejected in all cases except for the experienced EWES treatment.

22 However, if instead, we only use \(s_{it}\) as a regressor, it is highly significant. Regressing vote on SZ alone, or on PS alone, neither variable is statistically significant at the 5%.
outcomes between the two treatments. In contrast, GL predicts that the change in nominal bargaining power will sharply increase shares to the 45-vote block, and that the 9-vote block will always be part of the winning coalition. For inexperienced voters, the change in nominal bargaining power does, in fact, result in larger requests by those holding 45 votes vs. those holding 9 votes (a 0.53 share vs. a 0.48 share; \( p=0.05 \), two-tailed Mann–Whitney test). Note, however, that these requests by 45-vote blocks are no larger than the average share requested under the EWES treatment (a 0.55 share). Furthermore, these differences between 45-vote requests and 9-vote requests are no longer present for experienced subjects (0.52 for subjects with 9 votes and 0.53 for subjects with 45 votes). For inexperienced voters, 45-vote blocks offer shares to 9-vote blocks slightly more often than to 45-vote blocks (64% vs. 56%), but this difference is not significant at conventional levels.

In all other dimensions, behavior is the same across treatments. In particular, we cannot reject a null hypothesis that the fraction of MWCs is the same across treatments. Nor can we reject a null hypothesis that shares offered to 9-vote vs. 45-vote blocks are the same.

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23 The tests reported for this conclusion and the next one use all proposals, selected or not, and for all rounds within a given election.
24 These percentages sum to greater than one because of super majorities. To avoid repeated measures problems, these averages are calculated using subject averages as the unit of observation. These do not coincide with the population averages because some subjects play more rounds than others. A sign test is performed to establish if the percentage of offers to 9-vote blocks is the same as to 45-vote blocks.
Finally, the hypothesis that voting behavior is the same across both treatments cannot be rejected (even at the 10% level). 25

**Conclusion 6.** Under the UWUS treatment, 45-vote blocks offer coalition membership to 9-vote blocks significantly more often than to 45-vote blocks. Furthermore, as predicted under the symmetric BF model, a null hypothesis that 9-vote blocks are included in 90% of all such proposals cannot be rejected at conventional levels for experienced voters. For inexperienced 45-voter blocks, 9-vote blocks are included as coalition partners more often than 45-vote blocks (74% vs. 51%, \( p < 0.05 \), one-tailed sign test). This difference increases for experienced voters (77% vs. 47%, \( p < 0.1 \), one-tailed sign test). Even more striking, for experienced voters, we cannot reject (even at the 10% level) the null hypothesis that 9-vote blocks are given money nine times more often than 45-vote blocks. 26 That is, we cannot reject that proposers mix in the proportions predicted under the BF model.

Although this failure to reject the null hypothesis could be due to a combination of small sample size and the low power of the sign test, there is sufficient power to reject the null hypothesis that both types are equally likely to be invited into a coalition.

**Conclusion 7.** With experienced subjects, there are no significant differences in terms of proposed shares and voting behavior between the UWUS and the EWES treatments, nor between blocks of different size within the UWUS treatment, consistent with the BF model’s prediction. There are some minor differences in voting behavior for inexperienced subjects.

Shares requested by 45-vote blocks are not significantly different from those requested by 9-vote blocks for either inexperienced or experienced voters. (9-vote blocks average

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25 Interacting the share offered (\( s_i \)) with a dummy for the number of votes controlled by a subject (9, 33, or 45), we estimate the unrestricted model. Then a likelihood ratio test is performed using regression (5) as the restricted model.

26 Once again this is not a result at the population level, but rather each subject tends to mix in these proportions.
0.53 when inexperienced and 0.47 with experience, while 45-vote blocks average 0.55 and 0.53, respectively). Nor are they different from the shares requested under the EWES treatment (0.55 without experience and 0.56 with experience). As already noted, shares required to accept a proposal are greater for 45-vote blocks than for 9-vote blocks for inexperienced voters, but not experienced voters.

5. Discussion of results for $\delta=1$ treatments

Our decisive rejection of the comparative static predictions of GL is rather surprising given its robustness in field data (Browne and Franklin, 1973; Browne and Frendreis, 1980; Warwick and Druckman, 2001). The claim of support for GL using field data is based in large measure on using the proportion of seats held within the winning coalition as a regressor to explain the fraction of ministerial positions a party holds. Table 6 reports regressions similar to this for our data, where we substitute payoff shares for ministerial positions as the dependent variable. We limit ourselves to treatments EWES and UWUS because they both employ nominally proportional selection rules for recognizing proposals, the pattern found with respect to government formateur rules in field data (Diermeier and Merlo, 2004). The results clearly show that the percentage of votes controlled affects the share of the benefits received.

How can we reconcile the results in Table 6 with the rather decisive rejection of GL reported earlier? The key factor at work is the nominal proportional selection rule, in conjunction with the fact that proposers take larger shares for themselves and include the small voting block more often than the large block. Consequently, blocks with more votes wind up, ex post, taking more on average because they are selected to be the proposer more often, and they give smaller shares to their coalition partner. The role of these factors is rather dramatically illustrated in the simulation reported in the last columns of Table 6. There, we have generated results for 198 simulated elections (half for the EWES and half for the UWUS treatment) in which the simulated subjects strictly follow the SSPE of the BF model. The simulations (1) provide a close match to the experimental data, although

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27 That is, we regress the share allocated to a subject on the votes controlled by that subject divided by the number of votes in the winning coalition. Of course, to appropriately estimate such a model, we should account for the panel structure of the data. However, our intention is to reproduce the kind of estimation performed on field data, which do not correct for repeated observation, so we do not either.
our simulated voters are following the SSPE exactly, whereas real subjects do not, and (2) are consistent with a proportional relationship between payoff shares and the proportion of seats held by the winning coalition, consistent with GL.  

Note however, that the empirical results using field data are stronger than those reported in Table 6. Field data estimates yield a constant much closer to 0, the coefficient estimate for the percentage of votes held by the voting blocks is much closer to 1, and the \( R^2 \) is much higher. This superior fit of GL rests on the fact that real bargaining power is likely to be closely correlated with the number of seats controlled because the number of voting blocks (parties) is much larger than 3 in many countries. When there are many parties, the number of seats controlled provides a reasonably close approximation to real bargaining power most of the time. In any case, the lesson from the exercise reported in Table 6 is that even when GL is clearly violated in favor of the BF model (as in our simulations), the proportional selection rule yields a regression outcome consistent with GL.

The major quantitative deviation from the SSPE of the BF model rests on the significantly smaller share the formateur obtains relative to the model’s prediction (a 0.55 share conditioning on MWCs vs. the 0.67 share predicted). There are a number of candidate explanations for this failure of the model’s prediction. One is that because elections do not always end in round 1 formateurs, fearing retaliation in subsequent rounds, provides more generous offers. We doubt this is the case, however, because (i) a substantial majority of proposals are for MWCs, which is unlikely to be the case if retaliation in subsequent rounds was an issue and (ii) for elections that go beyond one round, we find no evidence that subjects exhibit either positive or negative reciprocity toward proposers in previous rounds in terms of the frequency with which these proposers are invited into subsequent coalitions.  

This last outcome is consistent with the stationarity assumption underlyng the SSPE refinement. Rather, what appears to be at work is a breakdown of the subgame perfect equilibrium prediction, in conjunction with the assumption that only own money payoffs matter, much like the breakdown reported in the experimental literature on bilateral bargaining games (see Roth, 1995 for a review of this literature).

These bilateral bargaining game results can help explain why proposer’s do not take as much as predicted, but only if the minimum threshold for responders is above 1/3 of the

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28 We also estimated two of the alternative specifications offered in the literature (e.g., Warwick and Druckman, 2001) using our simulated data: \( \bar{s}_{it} = 0.827 \times (\text{proportion of Votes Held})_{it} (R^2=0.865) \) where the constant has been dropped and \( \bar{s}_{it} = 0.791 \times (\text{proportion of Votes Held})_{it} + 0.043 (\text{proportion of Votes and proposer})_{it} (R^2 = 0.865) \) where the second variable interacts the proportion of seats held with a dummy variable for the proposer. The coefficient estimate on the proportion of votes held regressor is statistically significant in both specifications, but the coefficient estimate of that regressor interacted with the proposer dummy is not. Thus both specifications are consistent with GL. The statistical insignificance of proportion of seats held when it is interacted with a proposer dummy can be explained by the confounding effect of the proportional selection protocol.

29 For elections that have more than one round, we compute the number of times a subject includes the proposer from the previous round in his coalition and the number of times he does not. If proposers randomize between rounds the proposer from the previous round should be included as often as the other voter. Using a sign test and individual subjects as the unit of observation, we cannot reject a null hypothesis of randomization between rounds.

30 McKelvey (1991) conjectures that subjects may offer too much to potential coalition partners because of fear of retaliation in successive rounds (a nonstationary equilibrium).
pie. However, the 1/3 threshold is what one might expect based on: (i) the limited three-
player ultimatum game results reported (Guth and van Damme, 1998; Kagel and Wolfe,
2001); (ii) models attempting to explain behavior in the ultimatum game and related
experiments (Bolton and Ockenfels, 2000); and (iii) results from other legislative
bargaining game experiments (FKL). In fact, the 1/3 threshold provides a good ex-ante
predictor of average voting behavior in our games: Inexperienced (experienced) voters
reject 97% (98%) of all offers below 1/3 and accept 78% (89%) of all offers above 1/3.
(Offers of exactly 1/3 are accepted close to 50% of the time.)

The key to reconciling the reasonably good performance of the 1/3 rule of thumb
(ROT) with the failure of proposers to obtain shares close to the SSPE prediction rests on
the fact that for shares between 1/3 and 1/2, the rejection rate is relatively high (recall Fig.
1); sufficiently high that it is not profitable, in an expected value sense, to make offers
closer to the SSPE prediction. Rather, calculations suggest that in order to maximize
expected payoffs in a single round, proposers should take shares for themselves of
between 0.5 and 0.55 which is very close to what is observed.

The fact that voters mix in the correct proportion in the UWUS treatment is quite striking,
and at odds with other reported tests of mixed strategy equilibria (see, for example, Ochs,
1995; Erev and Roth, 1998). Note that, if we are willing to rely on more stringent
distributional assumptions, we can reject the null hypothesis of mixing in the correct
proportion using a $t$-test ($p$-value=0.05). Subjects do not mix in exactly the right
proportions, but they alter their behavior in the correct direction. One peculiarity of the
present design is that in equilibrium payoff shares do not change, only coalition membership
is affected. In contrast, in most other games with mixed strategy equilibria, subjects are
required to choose the higher payoff alternative less often in order to change the mix in the
required direction, which is counter-intuitive to subjects.

6. Results for $\delta=0.5$ treatment

Conclusions 1–4 carry over to the case with discounting.

(1) Most elections end in round 1: 89% (95%) for inexperienced (experienced) subjects.
These are higher percentages compared to EWES with $\delta=1$ ($p<0.05$, Mann–
Whitney test), suggesting that discounting increases the probability that elections
will end in round 1.

(2) Most proposals are for MWCs: 43% (77%) for inexperienced (experienced) voters.
However, if we consider offers that allocate a (1/30) share or less to the third player
to be approximate MWCs, then 62% and 85% of all offers are approximate MWCs
for inexperienced and experienced subjects, respectively.31

(3) Proposers take significantly larger shares for themselves than for the next largest
share holder: an average share of 0.50 (0.59) when inexperienced (experienced)

31 With $\delta=1$, distinguishing between strict MWCs and approximate MWCs has no impact as offers less than or
equal to (1/30) were quite rare.
compared to shares of 0.41 (0.39) for the second-highest shareholders ($p<0.01$, two-tailed sign test).

(4) Voting behavior is once again centered around 1/3 as the cut-off for accepting versus rejecting offers. Random effects probits yield results similar to those reported in Table 5 for the $\delta=1$ case: Own share is the only significant variable in the regression. However, in this case shares between 1/3 and 1/2 are significantly more likely to be accepted than when $\delta=1$.

**Conclusion 8.** Subject in the $\delta=0.5$ treatment accept, on average, lower shares than in the $\delta=1$ treatment.

The willingness of coalition partners to accept lower shares with $\delta=0.5$ reduces the income maximizing share proposers offer to 0.39 (0.36) for inexperienced (experienced) subjects. These offers would yield average shares of .61 (.64) for inexperienced (experienced) proposers, substantially larger shares than actually realized (shares of .50 and .59 for inexperienced and experienced proposers). The implication is that proposers were reasonably far away from maximizing their returns. However, a closer look at the data shows considerable adjustment over time in shares offered for the $\delta=0.5$ treatment, but not for $\delta=1$. In the $\delta=1$ treatments, proposals were almost immediately at the (expected single round) income maximizing level given how coalition partners voted. With $\delta=0.5$, proposers start out offering too much, but adjust their offers overtime so as to increase own shares. The result is substantially larger own shares for proposers in later periods, shares that were almost at the (expected) income maximizing level by the end of the experienced subject session.32

**Conclusion 9.** Proposer’s share increases with increases in the discount factor, as coalition partners are willing to accept smaller shares. However, it takes time for proposers to realize this. Furthermore, coalition partners appear unwilling to accept shares much below 1/3 and/or proposers are reluctant to make such low offers. As a result, proposer’s share is further away from the SSPE with $\delta=0.5$ (5/6) compared to the $\delta=1$.

The average own share for proposers in the last three elections of the experienced voter session with $\delta=1$ is 0.55 compared to 0.61 with $\delta=0.5$ ($p<0.05$, two-tailed Mann–Whitney test). This is a difference of 0.06 compared to the predicted difference of 0.167.33 Thus, lowering the discount factor affects behavior in the right direction: immediately lowering the acceptance threshold for coalition partners and slowly increasing proposer’s share. However, the threshold for accepting an offer does not dip below the (1/3) ROT suggested from the bilateral bargaining experiments (Bolton and Ockenfels, 2000). Potential

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32 For instance, averaging by individuals proposer’s own share in elections 1–7 vs. 8–10, using a sign test, we can reject the null hypothesis that they are the same against the one sided alternative that the shares are greater at the end of the sessions for both experience levels in the $\delta=0.5$ condition. On the other hand, the same test cannot reject the null with $\delta=1$.

33 As noted, proposers are still converging to the optimal share with $\delta=0.5$. However, even if they got to the share maximizing their expected return (0.64), the difference from the $\delta=1$ treatment would be 0.09 compared to the predicted difference, under the SSPE, of 0.167.
coalition partners’ reluctance to accept offers much below $1/n$ poses a fundamental barrier to achieving the SSPE in legislative bargaining games.

7. Conclusion

We have investigated the effect on legislative bargaining of changes in voting blocks’ nominal bargaining power, in the proposal selection protocol, and in the discount factor that applies when passage of legislation is delayed. These changes in treatment conditions permit us to distinguish the predictions of the BF model from the baseline of Gamson’s Law. The paper makes three basic contributions.

First, it improves our understanding of the performance of the BF model. All the comparative static predictions of the model find some support: Changing the number of votes each legislator controls without altering their real bargaining power has no effect on the distribution of payoffs between voting blocks, contrary to the predictions of GL, but consistent with BF. Changing nominal voting power but keeping the proposal recognition rule equal across legislators has no significant effect on the frequency with which the small voting block is invited into the winning coalition, consistent with the BF model and contrary to GL. In contrast, changing nominal voting power and changing the proposal selection rule to match the number of votes each legislator controls significantly increases the probability that the small voting block will be invited into the winning coalition, as both BF and GL predict. In all treatments, proposers obtain a larger share of the spoils than any other coalition member, as the BF model predicts, but this share is far smaller than predicted. All of these results are achieved with no discounting and a potentially infinite number of bargaining periods, so that they clearly result from the fundamental force that BF predicts drives behavior: potential exclusion from the winning coalition. Finally, introducing a discount factor between rounds decreases the share potential coalition partners need to accept a proposal and, with some lag, increases the share proposers ask for, consistent with BF. (GL is silent on this point.)

Second, the experiment has some implications for the growing social utility model literature in economics. Our results suggest that, in bargaining situations, subjects do not care for others in an altruistic sense, or in terms of maximizing the worst off players share, as own share is the only factor that affects voting. As in bilateral bargaining game experiments, proposers fail to take as much as predicted under the subgame perfect equilibrium, but come close to maximizing own expected return conditional on the heterogeneity in responders’ behavior. The $1/n$ rule of thumb found to organize behavior in Bolton and Ockenfels (2000) (also see Fréchette et al., 2003a) seems to act as a strict lower bound on acceptable offers both with and without discounting. However, it appears to take some discounting for it to serve as an acceptable rule of thumb for offers greater than $1/n$ (and less than $1/2$) within minimal winning coalitions.

Third, the experiment improves our understanding of previous empirical work using field data. Distinguishing between competing hypotheses such as GL and BF is not as simple as it might seem at first glance. Even within our simple experimental design, we find treatment conditions where the predictions of the two hypotheses coincide (EWES), and treatment conditions where GL predicts even stronger proposer power than BF. Our
results also suggest that the empirical findings of proportionality (between the percentage of seats political parties hold and the percentage of ministerial positions obtained) result in part from the fact that the role of government formateur is commonly associated with the party with the largest number of seats in parliament.

There are a number of potentially important complementary questions to address in future research. What happens with changes in real as opposed to nominal bargaining power? What is the impact of preproposal communication (cheap talk) that permits proposers to establish competition between potential coalition partners? This would seem to be part of any real world legislative bargaining process, and might well move proposer power closer to the BF predictions as proposers seek out the cheapest coalition partners. What will be the impact of veto players on outcomes (see Winter, 1996 for predictions within the BF framework)? These and a number of other interesting and important questions remain to be investigated.

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Appendix A

Proof of Proposition 1.


(II) Suppose first that there is one player—say player \(i\)—who is offered \(x < (1/3)\) in a SSPE, and the other players have an acceptance threshold strictly greater than \(x\). Then, any \(j \neq i\) would strictly prefer to offer \(x\) to \(i\) rather than offering to the other potential responder. But then, this means that the continuation payoff is \(x = \rho_i X + (1 - \rho_i)x\), where \(X\) denotes his payoff when he is a proposer. Note immediately that this implies \(X = x < (1/3)\). But then, consider any \(j \neq i\). Given the assumptions made, he can be offered \(y > x\), but he is only offered \(y\) when \(i\) is the proposer, since \(k \neq j \neq i\) prefers \(i\) to \(j\). Hence, \(y = 1 - x\). However, the continuation equilibrium equation requires \(y = \rho_j (1 - x) + \rho_j y = (1 - x)(\rho_j + \rho_i)\), which is in contradiction with \(y = 1 - x\). A similar logic allows to find a contradiction to the possibility that two players have equal expected payoff but both below \((1/3)\). Intuitively, in this
case, the third player would never be chosen as a responder, so the continuation equilibrium payoff would have to be low, contradicting his highest expected payoff.

(III) Given (II), the continuation equilibrium equations require:

\[
\begin{align*}
\frac{1}{3} &= \rho_i \frac{2}{3} + \rho_j \left( 1 - p^k_j \right) \frac{1}{3} + (1 - \rho_i - \rho_j) \left( 1 - p^i_k \right) \frac{1}{3} \\
\frac{1}{3} &= \rho_j \left( 1 - p^k_i \right) \frac{1}{3} + \rho_i \frac{2}{3} + (1 - \rho_i - \rho_j) p^j_i \frac{1}{3} \\
\frac{1}{3} &= \rho_i p^k_i \frac{1}{3} + \rho_j p^j_i \frac{1}{3} + (1 - \rho_i - \rho_j) \frac{2}{3} 
\end{align*}
\]

The solution to this system allows us to find the range for those three probabilities compatible with the unique acceptance threshold equal to (1/3). For every \( p^k_i \in [0,1] \), the other two probabilities have to be \( p^j_i = (\rho_i - p^k_i + p^j_k + p^k_i \rho_i + p^k_j \rho_j) / (\rho_j) \) and \( p^j_k = -(1 - \rho_i - 2 \rho_j - p^k_i + p^j_k + p^k_j \rho_j) / (\rho_i) \).

Notice that:

\[
\begin{align*}
0 \leq \frac{\rho_i - p^j_i + p^k_i \rho_i + p^k_j \rho_j}{\rho_j} \Rightarrow \left\{ \frac{p^j_i}{\rho_j} \leq \frac{\rho_j}{1 - \rho_i - \rho_j} \right\}, \\
0 \leq \frac{1 - \rho_i - 2 \rho_j - p^k_i + p^j_k + p^k_i \rho_i + p^k_j \rho_j}{\rho_i} \Rightarrow \left\{ \frac{p^j_i}{\rho_i} \geq \frac{1 - \rho_i - 2 \rho_j}{1 - \rho_i - \rho_j} \right\}, \\
1 \geq \frac{\rho_i - p^j_i + p^k_i \rho_i + p^k_j \rho_j}{\rho_j} \Rightarrow \left\{ \frac{p^j_i}{\rho_j} \geq \frac{\rho_i - \rho_j}{1 - \rho_i - \rho_j} \right\}, \quad \text{and} \\
1 \geq \frac{1 - \rho_i - 2 \rho_j - p^k_i + p^j_k + p^k_i \rho_i + p^k_j \rho_j}{\rho_i} \Rightarrow \left\{ \frac{p^j_i}{\rho_i} \leq \frac{1 - 2 \rho_j}{1 - \rho_i - \rho_j} \right\}.
\end{align*}
\]

Consequently, any profile of probabilities such that (I) holds constitute a mixed strategy SSPE. □

References


