PSYCHOLOGICAL EXPECTED UTILITY THEORY AND ANTICIPATORY FEELINGS*

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We extend expected utility theory to situations in which agents experience feelings of anticipation prior to the resolution of uncertainty. We show how these anticipatory feelings may result in time inconsistency. We provide an example from portfolio theory to illustrate the potential impact of anticipation on asset prices.

I. INTRODUCTION

We all experience feelings related to our uncertainty about the future, such as hopefulness, anxiety, and suspense. Psychologists have long recognized the importance of these anticipatory emotions, with anxiety theory being one of the most dynamic fields of psychological research.

Economists, in contrast, have paid little attention to the anticipatory emotions, and what attention has been paid has been limited to the case of a certain future, as in Jevons [1905] and Loewenstein [1987]. Loewenstein builds a model to explain why one might bring forward an unpleasant experience to shorten the period of dread, yet delay a pleasant experience in order to savor it. Since his model is deterministic, it is ill-suited to capturing anticipatory emotions, such as anxiety, that are predicated on an uncertain future.

In this paper we provide a new model of decision making under uncertainty in which we allow for a quite general class of anticipatory emotions. While our general theory is relatively unstructured, it nevertheless delivers novel results in almost all areas of application, in part due to the time inconsistency of individual preferences. Time inconsistency arises naturally in the presence of anticipation. As time passes, so do anticipatory emotions, and preferences may change as a result.

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1. Elster and Loewenstein [1992] provide a rich, yet informal, discussion of the importance of utility from anticipation, as well as utility from memory.

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We begin in Section II with a brief and highly selective survey of some of the relevant psychological literature on anticipation. We focus our discussion on the nature and determinants of anxiety, since this has been a particularly active area of research. For our purposes, the most important conclusions are that anxiety is anticipatory and that the desire to reduce anxiety motivates many decisions.

In Section III we show how to extend the expected utility model to incorporate anticipation. Our approach is to expand the standard prize space to include anticipatory emotions such as suspense and anxiety, and to model how lotteries over future physical prizes influence these emotions. We then place our preferences into a two-period decision problem, and prove that optimal strategies exist. Because we expand the state space to include states of mind, we refer to our model as the psychological expected utility model.

In Section IV we specialize the model and focus on the portfolio decisions of an anxious saver. We argue that the incorporation of anxiety into asset pricing models may help explain both the equity premium puzzle and the risk-free rate puzzle. Safe assets, by providing secure returns, may reduce anxiety even before final consumption takes place. They therefore provide an extra benefit in addition to the smoothing of final consumption across states, serving to reduce the risk-free rate. Stocks and other risky assets, however, by increasing the variance of the portfolio, tend to increase anxiety in the period before final consumption takes place. Hence owning stocks involves an extra cost in addition to increasing the variance of final consumption, which increases their required return.

Anxiety complements risk aversion in our discussion of the risk-free rate puzzle and the equity premium puzzle. The discussion in Section IV, however, also points out several differences between anxiety and risk aversion. One is that anxiety may respond more directly to possibilities than to probabilities so that agents may appear to “overreact” to small probability events. Another is that anxiety is an anticipatory emotion, whereas risk aversion is a static concept. Hence there will be a time profile to anxiety that is absent in attitudes toward risk. Finally, we discuss the interaction between anxiety and attention, and show how agents may avoid anxiety by putting their heads in the sand with the obvious consequences for the quality of decision making.

Section V discusses other applications and extensions of the
framework. Section VI discusses the relationship between our work and prior work in decision theory. Section VII briefly discusses some of the policy implications of the model. Section VIII contains concluding remarks.

II. AVERSIVE FEELINGS ABOUT FUTURE LOTTERIES: THEORY AND EVIDENCE

To motivate our model, we present evidence from the psychological literature on the anxiety, worry, and fear constellation of emotions. We first outline qualitative theories of anxiety, and discuss the experimental evidence on the determinants of anxiety. We then present evidence that people make choices that are aimed at lowering their level of anxiety.

II.A. Determinants of Anxiety

Two agreed principles of anxiety theory are that the emotion is *anticipatory* and *aversive*. According to the glossary of the Diagnostic and Statistical Manual of Mental Disorders [American Psychiatric Association 1987], the term anxiety denotes “apprehension, tension, or uneasiness that stems from the anticipation of danger” [p. 392]. Recent work on worry is even more clear-cut in this regard:2 “Worry is a cognitive phenomenon, it is concerned with future events where there is uncertainty about the outcome, the future being thought about is a negative one, and this is accompanied by feelings of anxiety” [MacLeod, Williams, and Bekerian 1991, p. 478].

Research indicates that these anticipatory feelings can be acute. Andrykowski, Redd, and Hatfield [1985], for example, document that many patients who are due to receive chemotherapy experience bouts of vomiting and related noxious symptoms during the 24 hours *prior to* treatment. In some cases the anticipation of an event may be worse than the event itself. Lazarus [1966] provides a survey of experiments demonstrating that certain forms of physical pain, such as pinpricks, do not produce measurable psychological-stress reactions beyond those produced by the mere anticipation of such conditions.3

2. As Eysenck [1997] points out, Aristotle was already clear on the subject: “Let fear then be a kind of pain or disturbance resulting from the imagination of impending danger, either destructive or painful” [Aristotle in Eysenck].

3. Whereas much anxiety is purely anticipatory, there is also evidence that anxiety can be learned. In their study of patients' levels of anxiety prior to their
It is now widely believed that the anticipatory and unpleasant nature of fear are two aspects of an innate mammalian “fear system.” Many theorists view fear as a “basic emotion” with a profound evolutionary role in species preservation. In work with both rats and humans, LeDoux [1993] has established the important role of the amygdala in the preconscious evaluation of a stimulus as representing a potential threat. This evaluation serves to trigger a set of internal and external responses that aid in the further evaluation of the frightening stimulus, with conscious feelings of fear and anxiety representing but one part of this overall system. In this vision, the aversive nature of anxiety is connected to the appropriate “flight” response.

The fact that there are important neurological and physiological concomitants of anxiety has proved very important in establishing higher measurement standards in the field of anxiety research than in most other branches of psychological theory. In the early experimental literature, skin conductance and pulse rate were among the key measures used, in addition to self-reported data. The arsenal of available measures is currently undergoing rapid expansion, as experimentalists manipulate the environment to induce fear, and then measure as many aspects as possible of the bodily and neurological response.

II.B. Anxiety and Choices

Experimentalists have uncovered many regularities in the determinants of anxiety. One is that an extended waiting period before a stressful event results in a markedly greater buildup of anxiety than does a shorter waiting period [Nomikos et al. 1968]. This suggests that it may be better to undertake an anxiety-inducing task immediately in order to shorten the duration of the anxiety. A number of studies support this intuition. Cook and Barnes [1964] performed some early experiments linking feelings

first six doses of chemotherapy, Jacobsen, Bovberg, and Redd [1996] are interested in the dynamic pattern of anxiety. Their results are broadly supportive of a simple model of learning theory, whereby a bad experience in an earlier chemotherapy session results in elevated anxiety prior to subsequent sessions.

4. The origin of this work is Darwin’s theory of mammalian facial expressions as defining of emotions. In support of this, it has been found that there is tremendous cross-cultural agreement about which facial expressions convey fear, as opposed to other emotions, such as elation.

5. See Gerritsen et al. [1996] for an application of this methodology to social fear, and Cacciopo et al. [1993] for a rich discussion of studies that attempt to distinguish fear responses from other emotional responses.

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of anxiety to choice behavior. They offered subjects a choice between a large immediate electric shock and a lesser shock that would be delayed by eight seconds. They found that many subjects chose the larger shock rather than waiting anxiously for the smaller shock. A similar pattern was identified by Loewenstein [1987], who used a survey technique to confirm that many subjects would prefer an immediate electric shock over a shock delayed by 24 hours.

There has also been a significant volume of research on whether additional information about an upcoming event will serve to raise or to lower anxiety. The pioneering work in this area is due to Janis [1958], who believed that providing information prior to a medical procedure would stimulate the “work of worrying” that would initially raise anxiety, but subsequently lower anxiety (and possibly speed recovery). Consistent with this theory, Klusman [1975] reported results of an experiment in which childbirth information generally lowered anxiety, and lowered patient self-reports of pain.

It is now widely accepted that not all individuals want or benefit from information, and that information actually serves to raise anxiety in some cases. In one of a number of similar studies, Miller and Mangan [1983] performed an experiment on subjects who were about to undergo a stressful medical procedure (colonoscopy). The patients first completed a questionnaire upon the basis of which they were categorized as either “blunters” or “monitors.” Blunters answered the survey in a manner suggestive of information avoidance, while monitors’ answers suggested a pattern of information gathering. Voluminous prior information about the procedure was then provided to half of the monitors and half of the blunters, with minimal information given to the others. The results confirmed the theory that more information lowered the anxiety level of the monitors, but raised the anxiety level of the blunters.6

Given that information influences anxiety, it is not surprising that people often collect information in a way that reduces anxiety. The desire of some subjects to avoid information in anxiety-inducing situations was confirmed experimentally by

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6. The discovery of widely divergent anxiety responses to information is part of a broader analysis of differences in anxiety responses across individuals. The distinction between anxiety-inducing external states of the world, and anxiety-proneness as an individual trait has been central in the recent theoretical literature [Spielberger 1972].
Averill and Rosenn [1972] (see Section V below for details). More recently, these findings have moved out of the laboratory and into the field. In a study by Lerman et al. [1998], subjects who had earlier given a blood sample were told that their blood sample had been analyzed to identify whether or not they were carriers of one of two genetic mutations that indicate susceptibility to breast cancer later in life. Subjects were then asked whether or not they wanted to know the result of the test. Despite the clear planning advantage of knowing one’s status, a large set of experimental subjects reject this information. Of the 396 individuals who were included in the study, 227 chose to receive the information, while the remaining 169 declined.

II.C. Toward a Decision-Theoretic Framework

The psychological research on anxiety needs to be placed in a decision-theoretic setting in order to ask and answer the many important policy questions that are raised. The framework must respect the qualitative findings that anxiety is anticipatory and aversive. Where possible, the level of anxiety should be related to beliefs in a manner that fits with the experimental evidence, and the theory should therefore have no difficulty in matching the findings concerning how anxiety influences choices. Finally, the model should allow us to develop decision-theoretic and game-theoretic approaches in order to identify welfare-enhancing policies in settings in which anxiety matters. Our model is designed to accomplish these broad objectives not only for the case of anxious feelings, but also for the broader class of anticipatory emotions.

III. The General Framework

The model has three components. The first is a definition of the relevant prize space. We replace the standard prize space with a space of “psychological states,” comprising a complete (for model purposes) description of the individual's state of mind. An individual can experience many different mental states such as anxiety or excitement, and it is these that we connect to the agent’s level of utility and well-being. The second component of the model is a formal description of the space of lotteries in the physical world, and of the manner in which uncertainty about these lotteries resolves over time. The final component is a mapping that connects physical lotteries with mental states. We refer
to the complete model as the *psychological expected utility* model. After presenting the basic model, we develop the corresponding decision-theoretic framework and prove that optimal strategies exist.

### III.A. The General Model of Individual Preferences

We model an agent’s state of mind as a vector of real numbers. Formally, there are two periods and two spaces, \( X_t \subseteq \mathbb{R}^{n_t}, \) \( t = \{1,2\} \), that represent the possible psychological states in each of the two periods. Let \( X \) denote the product space \( X_1 \times X_2 \). The space of psychological lotteries \( P(X) \) is the space of all Borel probability distributions on \( X \). We endow \( X \) with the product topology and \( P(X) \) with the topology of weak convergence. Since each space \( X_t \) is separable, \( X \) is separable, and \( P(X) \) is separable and metrizable (by the Prohorov metric).

Note that \( P(X) \) is a mixture space: for \( \lambda \in [0,1] \) and \( p, q \in P(X) \), there exists an element of \( P(X) \), which we shall denote \( p \lambda q \), that assigns to each Borel measurable subset \( A \subseteq X \) the probability \( \lambda p(A) + (1 - \lambda)q(A) \).

We assume that the decision maker has a preference relation \( \succeq \) defined on psychological lotteries \( P(X) \) that satisfies standard axioms for choice under uncertainty, including a substitution axiom.

**Assumption 1.** Make the following assumptions on \( \succeq \):

(i) \( \succeq \) is complete and transitive;
(ii) for all \( p, q, r \in P(X) \) and \( \lambda \in [0,1] \), \( p \succeq q \) implies \( p \lambda r \succeq q \lambda r \);
(iii) given \( p, q, r \in P(X) \) such that \( p \succeq q \) and \( q \succeq r \), there exist \( \lambda_1, \lambda_2 \in [0,1] \) such that \( p \lambda_1 r \succeq q \) and \( p \lambda_2 r \preceq q \);
(iv) for all \( p, q \in P(X) \), \( (p_1, p_2) = (q_1, q_2) \) implies that \( p \sim q \);
(v) \( \succeq \) is continuous in the topology of weak convergence.\(^7\)

Assumption 1(ii) is the substitution axiom, which we assume

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7. A binary relation \( \succeq \) defined on a mixture space \( \Omega \) is continuous if for all \( x, y, z \in \Omega \):

\[
\{ \lambda \in [0,1] : x \lambda y \succeq z \}
\]

and

\[
\{ \lambda \in [0,1] : z \succeq x \lambda y \}
\]

are closed.
holds on the space of psychological lotteries. Assumption 1(iv) is sufficient for the additive separability of preferences over time, which we assume for convenience.

Assumption 1 is sufficient to ensure that there exists an expected utility representation of $\succeq$ (see Fishburn [1982], Theorem 10.1 and p. 73). Let $E_{r}f$ denote the expectation of the random variable $f$ with respect to the measure $r$.

**Proposition 1.** Given Assumption 1, there exists a bounded, continuous function $U : X \to R$ such that for $p, q \in P(X)$, $p \succeq q$ if and only if $E_{p}U \geq E_{q}U$. In addition, the function $U : X \to R$ has a time additive representation,

$$U(x) = u_{t}(x_{1}) + u_{2}(x_{2}),$$

where $u_{t} : X_{t} \to R$, for $t \in \{1,2\}$.

Turning to the physical lotteries, the basic data are two spaces describing all of the physical prizes that the agent may receive in each period, $Z_{t} \in R^{m_{t}}$, $t \in \{1,2\}$. Since our concern is anticipation, the timing of physical and psychological lotteries and utility is important. We take as our convention that the period $t$ psychological state is realized at the end of period $t$, immediately after the physical state for the period is realized. Hence there is no time for anticipation within a period. Given the two-period setup, this means that the agent will only experience feelings of anticipation in the first period. Those feelings will only concern the second-period uncertainty that remains unresolved after the outcome of the first-period lottery has been realized. Since there are no anticipatory feelings in the second period, we make the simplifying assumption that the physical and psychological reward spaces are the same in the second period, $Z_{2} = X_{2}$. This is equivalent to assuming that only the actual physical prize in period 2 influences that period's psychological state.

The first period is more subtle. To capture anticipatory emotions, we allow the psychological prize in period 1 to depend not only on the physical prize received in the first period, but also on the remaining uncertainty concerning the physical prize that will be realized in the second period. To capture this, we follow the methods of Kreps and Porteus [1978] to formalize evolving uncertainty. Let $Z_{1}$ be the space of physical prizes in period 1, and $L_{2}$ the space of Borel probability distributions over period 2 prizes. Define $Y_{1} = Z_{1} \times L_{2}$, and let $L_{1}$ be the space of Borel
probability measures over $Y_1$. We endow $Y_1$ with the product topology, and $L_1$ with the topology of weak convergence. It can be shown that both $L_1$ and $Y_1$ are separable, metrizable spaces (see Kreps and Porteus [1978]).

The elements of $Y_1$ can be thought of as the pure outcomes in the first period; these pure outcomes include a prize from $Z_1$ and a lottery over future prizes from $L_2$. Each such outcome determines just how much uncertainty regarding the second-period prize remains to be resolved in the second period. Thus, a lottery $l_1 \in L_1$ specifies the period 1 view of the likely state of knowledge at the end of the period. In this way, a lottery $l_1 \in L_1$ encodes the timing of the resolution of uncertainty. Following Kreps and Porteus, we refer to each element $l_1 \in L_1$ as a \textit{temporal lottery}.

To complete our model of preferences, we define the function $\phi : Y_1 \rightarrow X_1$ which gives the psychological state that results from an agent facing the outcome $y_1 \in Y_1$. We assume that $\phi$ is continuous.

We are now in a position to define the utility function over temporal lotteries. This utility function is induced by the mapping $\phi$ and the utility function over psychological prizes. Given $y_1 = (z_1, l_2) \in Y_1$,

\begin{equation}
V_1(y_1) = u_1(\phi(y_1)) + E_{l_2}[u_2(x_2)].
\end{equation}

$V_1$ looks like a standard time-separable expected utility function except for the presence of $l_2$ in first-period utility. Given any two temporal lotteries $l_1, l'_1 \in L_1$, $l_1 \succeq l'_1$ if and only if $E_{l_1}V_1(y_1) \succeq E_{l'_1}V_1(y_1)$.

As we will see in Section IV, it is $\phi$ that gives the theory structure. It is in this mapping that we capture different psychological attitudes toward uncertainty.\footnote{The nature of the model as a formulation of anticipatory feelings is captured in the assumption that the period $t$ feelings are based purely on the current physical prize and uncertainty that is as yet unresolved. The past prizes and past psychological states play no role in determining current feelings.}

\section*{III.B. A General Decision Problem}

We consider decision problems with the following general structure. Given an initial state $s_1 \in S_1$, the agent chooses an action $\alpha \in A_1$ from a feasible set $\Gamma(s_1) \subseteq A_1$. The initial state and action determine the physical payoff in the first period according to a payoff function, $\eta : S_1 \times A_1 \rightarrow Z_1$. Between the first
and the second period, a new exogenous state $s_2 \in S_2$ is realized according to the Markov transition function $Q(B,s)$,

$$Q(B,s) = \Pr\{s_2 \in B | s_1 = s\}.$$ 

Finally in the second period, the agent chooses a lottery over second-period prizes from a feasible set that may depend both upon the earlier action choice and the new information, $\Gamma_2(\alpha_1, s_2) \subseteq L_2$. After this, all remaining uncertainty is resolved, and the second-period payoff is realized. We make standard simplifying assumptions on the various data of the problem.

**Assumption 2.** The spaces $A_1$, $S_1$, and $S_2$ are subsets of finite-dimensional Euclidean spaces. The choice correspondences $\Gamma_1$ and $\Gamma_2$ are compact valued and continuous. The function $\eta : S_1 \times A_1 \rightarrow Z_1$ is continuous in the second argument.

We define strategies in the standard fashion. A period 2 policy is a measurable function $\pi_2 : A_1 \times S_2 \rightarrow L_2$ with $\pi_2(\alpha_1, s_2) \in \Gamma_2(\alpha_1, s_2)$. We let $\Pi_2$ denote the set of such policies. An overall strategy is a combination of an initial choice $\alpha_1 \in \Gamma_1(s_1)$ and a second-period policy $\pi_2 \in \Pi_2$.

The lottery over second-period prizes that the agent anticipates at the end of period 1 depends both on the distribution of second-period exogenous shocks and on the strategy selected in the second period. We denote this lottery by $\lambda(\alpha_1, \pi_2 | s_1) \in L_2$,

$$\lambda(\alpha_1, \pi_2 | s_1) = \int \pi_2(\alpha_1, s_2) Q(ds_2, s_1).$$ 

Here $\pi_2$ gives the second-period prize distribution conditional on the second-period state. Integrating over second-period states yields $\lambda$.

The dependence of the first-period temporal lottery on the second-period strategy gives rise to the time inconsistency of optimal choices, and requires us to take a stand on the appropriate definition of optimal strategies in the presence of time inconsistency.

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9. Whether or not mixed strategies are allowed in period 2 is determined by the nature of the correspondence $\Gamma_2(\alpha_1, s_2)$. 
III.C. Optimal Strategies: Definition and Existence

Our approach to the definition of optimal strategies in the presence of time inconsistency is to exploit the recursive structure of the optimization problem. The appropriate standard of rationality in period 2 is straightforward: we know that the agent in the second period will make a decision that is optimal for that period alone. Let $J_2(\alpha_1, s_2)$ denote the value of an optimal policy conditional on the second-period state and first-period action. In the second period the agent chooses a lottery from $\Gamma_2$ that maximizes expected utility:

$$J_2(\alpha_1, s_2) = \max_{l_2 \in \Gamma_2(\alpha_1, s_2)} E_{l_2}[u_2(z_2)].$$

We define the second-period choice correspondence in the natural manner as

$$G_2(\alpha_1, s_2) = \{l_2 \in \Gamma_2(\alpha_1, s_2) | J_2(\alpha_1, s_2) = E_{l_2}[u_2(z_2)]\}.$$

An optimal policy in period 2 is a measurable selection from $G_2(\alpha_1, s_2)$. Let $\Pi_2^s$ denote the set of optimal period 2 policies. Note that with our assumptions, the theorem of the maximum and a standard measurable selection theorem guarantee that $\Pi_2^s$ is nonempty [Hinderer 1970].

The subtle point in the definition of optimal strategies concerns the nature of the period 1 choice. There may be more than one optimal policy in the second period, and the method of selecting among such indifferent policies may impact the payoff in the first period. Without time consistency, there is no presumption that indifference extends back from period 2 to period 1. Following Strotz [1955], we assume that the agent is able to select any future strategy provided that there is no future contingency in which there is some alternative strategy that is strictly preferred.

Given an initial state $s_1 \in S_1$, we refer to strategies $\pi = (\alpha_1, \pi_2)$ with $\pi_2 \in \Pi_2^s$ as consistent strategies. The (nonempty) set of such strategies is denoted $\Pi^C(s_1)$. Let $J_1(s_1)$ denote the value of an optimal strategy in the first period; then

$$J_1(s_1) = \sup_{\alpha_1 \in \Gamma(s_1)} \sup_{\pi_2 \in \Pi_2^s} u_1(\phi(\eta(s_1, \alpha_1), \lambda(\alpha_1, \pi_2 | s_1)) + E_{\lambda(\alpha_1, \pi_2 | s_1)}[u_2(z_2)].$$

An optimal strategy is a choice $(\alpha_1, \pi_2) \in \Pi^C(s_1)$ that maximizes...
A slight adaptation of a theorem of Harris [1985] shows that an optimal strategy exists.

**Proposition 2.** An optimal strategy exists.

*Proof.* If we consider the decision-maker in period 1 and in period 2 as two separate individuals, our solution concept is equivalent to selecting the subgame perfect equilibrium that is best from the period 1 perspective. It is easy to confirm that our model satisfies all of the assumptions of Theorem 1 in Harris [1985]. This Theorem states that the set of subgame perfect equilibria compact. Since our objectives are continuous, an optimum exists.

IV. ASSET PRICES AND ANXIETY

The model of Section III is relatively unstructured. In order to gain insight into the implications of anxiety, it is useful to consider a specific application. One area of economics in which we might expect anxiety to play an important role is portfolio choice. Below, we present a simple model and explore the implications of anxiety for asset pricing.

IV.A. The Model

Consider a two-period Lucas tree model of consumption and saving [Stokey and Lucas 1989, p. 300]. A representative agent is born with an endowment of a consumption good equal to $w_1$. The consumption good is nonstorable. The agent is also endowed with $N$ productive assets, each in fixed supply (normalized to unity), that yield random quantities of a consumption good in the second period. Asset $n \in N$ yields $s_n$ units of the consumption good in period 2.

In each period there are spot markets for the consumption good and for shares in the assets. Given the fixed supplies of goods and assets, the competitive equilibrium in this economy is trivial: first-period consumption is equal to the endowment, $c_1 = w_1$; second-period consumption is equal to the random output of the assets $c_2 = \Sigma s_n$; and the consumer willingly holds all of the assets. The question is what prices support this allocation.

Our point of departure from the standard model is that in addition to valuing consumption, the utility of the representative agent depends on the anxiety associated with holding risky assets. To make the connection with our model explicit, we define
the physical prize spaces to be $Z_1 = Z_2 = R$, corresponding to units of consumption, $c_t \in Z_t$. It follows that the space of temporal lotteries is the set of pairs $y_1 = (c_1, l_2) \in Y_1$, where $l_2$ represents a lottery over second-period consumption levels. The final component of the model is the mapping $\phi : Y_1 \to X_1$ describing the relationship between temporal lotteries and the mental state in period 1. We assume that $\phi(c_1, l_2) = (c_1, a(l_2))$, where $a : L_2 \to R$ is a differentiable function that measures the anxiety associated with the lottery $l_2 \in L_2$.

With this formalization, the induced expected utility function (1) takes the form,

$$V_1(c_1, l_2) = u_1(c_1, a(l_2)) + \beta E_{l_2}[u_2(c_2)].$$

In general, $l_2$ depends on the portfolio that the agent holds. Let $\theta$ denote the vector of portfolio shares held by the agent, and $\theta_n$ the share of asset $n$. All shares will be unity in equilibrium. Define $l_2(\theta)$ to be the distribution of the random variable $\sum_n s_n \theta_n$, and let $\tilde{a}(\theta)$ be the differentiable composite mapping $a(l_2(\theta))$ that associates with each portfolio the corresponding level of anxiety.

The agent chooses the level of first-period consumption and the asset portfolio to maximize utility subject to the budget constraint,

$$c_1 + \sum_n p_n \theta_n = w_1 + \sum_n p_n,$$

where $p_n$ is the price of asset $n$ in terms of the consumption good in period 1.

The first-order condition for asset $n$ is

$$\frac{\partial u_1}{\partial c_1} p_n = \frac{\partial u_1}{\partial a} \frac{d\tilde{a}}{d\theta_n} + \beta E_{l_2} s_n \frac{\partial u_2}{\partial c_2}.$$

By reducing consumption by $p_n$ units in the first period, the agent can purchase one unit of asset $n$, thereby raising consumption by $s_n$ units in the second period. The portfolio adjustment, however, also has an effect on the level of anxiety in the first period reflected in the first term on the right-hand side of (3).

Rearranging (3) and plugging in the equilibrium conditions pins down the price of the asset:

$$p_n = \frac{\frac{\partial}{\partial a} u_1(w_1, \tilde{a}(1))(\partial/\partial \theta_n)\tilde{a}(1) + \beta E_{l_2} s_n (\partial/\partial c_2) u_2(\sum_n s_n)}{\frac{\partial}{\partial c_1} u_1(w_1, \tilde{a}(1))},$$
where \( \mathbf{1} \) is the vector consisting of all ones. Since anxiety is aversive, \( \frac{\partial u_1}{\partial a} \) is negative. It is immediate that an asset that causes anxiety has a lower price and a higher required rate of return. We now discuss the properties of the anxiety function.

**IV.B. Anxiety, Risk, and the Equity Premium**

Suppose that anxiety is decreasing in the mean and increasing in the riskiness of second-period consumption.\(^{10}\) Suppose for simplicity that this relationship is linear:

\[
a(l_2) = -\alpha E_{l_2} c_2 + \gamma \text{var}_{l_2}(c_2),
\]

where \( \alpha \) and \( \gamma \) are positive parameters. Since \( c_2 = \sum s_n \theta_n \), it follows that

\[
\frac{d}{d\theta_n} \tilde{a}(1) = -\alpha E_{l_2} s_n + 2\gamma \text{cov}_{l_2}(c_2, s_n).
\]

The first term reflects the effect on anxiety of an increase in the holding of asset \( n \) through the mean of consumption, and the second term the effect through the variance.

In this formulation the presence of anxiety helps to explain both the risk-free rate puzzle and the equity premium puzzle. For a riskless asset, in which \( s_n \) is constant, \( \frac{d\tilde{a}}{d\theta_n} = -\alpha s_n < 0 \). It follows that the price of the riskless asset is greater than the price that it would take in the standard model. In this view, the agent is purchasing “peace of mind” along with the asset, and this justifies the low risk-free rate.\(^{11}\)

Since stocks are risky, their purchase will tend to increase both the mean and the variance of second-period consumption. The sign of \( \frac{d\tilde{a}}{d\theta_n} \) will depend on how these two effects balance out. If \( \gamma \) is sufficiently large relative to \( \alpha \), the effect through the variance will dominate, and \( \frac{d\tilde{a}}{d\theta_n} \) will be positive. In this case,

\(^{10}\) That anxiety would be decreasing in the mean and increasing in the variance of consumption is implicit in most characterizations of anxiety. For example, Lazarus \([1966]\) defines threat as the variable that induces anxiety, and concludes that the “degree of threat is a function primarily of amount, imminence, and likelihood of the anticipated harm” \([p. 43]\).

\(^{11}\) Hogarth and Kunreuther \([1995]\) found that people refer to peace of mind rather than probabilities when defending purchases of warranties. They tend only to think about probabilities if these figures are given to them.
anxiety will reduce the price of stocks and increase their return relative to the standard model. Here stock ownership entails psychic costs. The agent has to live with the anxiety that accompanies the holding of a risky portfolio.  

Many discussions of risk appear to confuse anxiety and risk aversion. We believe that it is important to distinguish between these two phenomena. Anxiety is an anticipatory emotion experienced prior to the resolution of uncertainty. It is related to the feeling of living with uncertainty. In contrast, risk aversion is a static concept pertaining to the curvature of the utility function within a period.

By ignoring anxiety, conventional measures of risk aversion underestimate the effect of uncertainty on asset prices. For example, the measure of risk aversion that Barsky et al. [1997] obtain from their survey concerns a static gamble. A typical question that they ask is whether agents would prefer a job that guarantees their current income for the rest of their life to a lottery between a job that will double their income and a job that will cut their income by a third. In terms of our model, they are learning only about the curvature of the utility function over second-period consumption levels, and not the anxiety that workers may associate with living with the possibility that their wage might change. As we have seen, such an approach will underestimate the equity premium.

**IV.C. Possibility not Probability**

The analysis above shows how an anxious individual may appear more risk averse. Anxiety, however, also differs from risk

12. Our explanation of the equity premium and risk-free rate puzzles shares important common elements with the work of Epstein and Zin [1989], Farmer [1990], and Weil [1990] based on application of the Kreps-Porteus model. In fact, the Kreps-Porteus model is a special case of our model in which preferences are time consistent. In addition to highlighting the role of anticipation and time inconsistency, the critical difference between the models lies in the assignment of utility to specific periods in our model. The lack of specificity concerning the assignment of utility across periods in the Kreps-Porteus model means that there are many different psychological expected utility models that reduce to the same Kreps-Porteus model. See Caplin and Leahy [1999] for more on this last point and an application to welfare.

13. For example, in a section titled “Your Attitude Toward Risk: A Questionnaire,” TIAA-CREF recommends a portfolio allocation on the basis of four questions, one of which is, “How would you rate your ability to tolerate the stress of investment volatility?” [TIAA-CREF 1999, p. 19].

14. Loewenstein et al. [1999] refer to an experiment by Weber and Clore who found that subjects who were made anxious through hypnosis appeared significantly more risk averse than a control group. These subjects appeared to associate their feelings of anxiety with the uncertainty of outcomes for the gamble options.
aversion in important ways. In particular, it may explain why agents appear to overreact to small probability events.

Harless and Camerer [1994], in a metastudy of laboratory experiments on risk taking, report that the expected utility model performs remarkably well if agents are choosing from gambles with the same support and quite poorly if the gambles are of different support. Behavior is generally consistent with the model when the probability of an event changes from 34 to 35 percent or from 50 to 51 percent, but deviates markedly if the probability changes from 0 to 1 percent.

The introduction of anticipatory emotions can help to explain this failure of the expected utility model. Damasio [1994] posits that anticipatory emotions arise in reaction to mental images of the outcome of a decision, and that these mental images are discrete. If this is the case, then changes in probability within some broad midrange of values may have little effect on anticipatory emotions and therefore little effect on the rankings of lotteries. Changing the probability of an outcome from zero to some small positive number, however, may have a large effect on anticipation.

This view finds empirical support in the work of Monat, Averill, and Lazarus [1972] and Bankhart and Elliot [1974]. In their experiments subjects were informed that they would receive a painful electric shock at a certain point in time with varying degrees of certainty. Measures of anxiety such as heart rate and skin conductance showed little sensitivity to the probability of the shock unless this probability was zero. In the words of Loewenstein et al. [1999], many decisions are sensitive “to the possibility rather than probability of negative outcomes.”

The implications for asset prices are immediate. If anxiety reacts to possibilities, then asset prices will tend to “overreact” to small probability events. For example, the announcement that the Federal Reserve is concerned with inflation, will focus investors’ attention on the possibility of an increase in interest rates. This will have an effect on investor anxiety independent of the likelihood of a rate increase.

**IV.D. The Time Course of Anxiety**

As mentioned above, anxiety is an anticipatory emotion, whereas risk aversion is a static concept. It follows that one of the

15. Loewenstein et al. [1999] discuss these differences extensively.
places that the two should differ is in their timing. The literature in experimental psychology suggests that once an individual is fully aware of an upcoming threat, the time path of anxiety tends to be U-shaped [Breznitz 1984]. There is intense fear when an individual is first informed of an upcoming threat. This fear then diminishes for a while, before rising sharply in anticipation of the impact. Moreover, an extended waiting period prior to a stressful event occurs results in a markedly greater buildup of anxiety than does a shorter waiting period [Nomikos et al. 1968].

These studies suggest a complex relationship between the time that a threat is recognized $T_0$, the time that a threat is expected to materialize $T^*$, the current time $t$, and anxiety $a$. All else equal, newly announced risks ($t = T_0$) should cause greater anxiety than recently announced risks. Moreover, the buildup prior to impact rises as $t$ approaches $T^*$ and is greater the longer the period of anticipation ($T^* - T_0$). There is no analogous role for $T_0$ in the analysis of risk.

**IV.E. Attention**

An experiment of Averill and Rosenn [1972] provides a graphic illustration of the connection between anxiety and attention. Subjects were told that there would be an electric shock coming, and that they could switch at will between listening to one of two channels of a tape recorder. One channel would broadcast a warning signal directly before the shock. The second channel played music, so that listening to it precluded hearing the warning signal. The subjects were divided into two groups. The first group was told that even if they heard the signal, they would be unable to avoid the shock. The second group was told that if they heard the warning signal, they would be able to avoid the shock if they immediately pressed a button. In the first group, 50 percent of the subjects chose to listen to the warning even though evasion was impossible. In the second group, 23 percent of the subjects chose to listen to the music, even though this made it inevitable that they would receive the shock!

The desire to “put one’s head in the sand” appears general. In summarizing the experimental literature, Miller [1987] writes: “The evidence indicates that the way in which individuals selectively attend to and process threat-relevant cues in a given situation determine how stressed and anxious they become in that situation” [p. 345].

The most direct asset market application concerns a possible
desire on the part of an anxious investor to avoid information. If this desire is important, it may dominate the possible planning advantages involved in paying detailed attention to the current state of the portfolio.

One area in which such avoidant behavior might have practical implications is saving for retirement. Bernheim [1995] offers a sobering assessment of the savings of the baby boom generation, arguing that most households are far from adequately prepared for retirement. He further argues that the low levels of accumulated wealth are unlikely to reflect standard life-cycle considerations, and are more likely to reflect psychological issues, and in particular psychological impediments to adequate planning.16

A vision of how anxiety and attention interact may aid in understanding planning problems in the period leading up to retirement. Thinking about all of the financial and life-style changes in the retirement period is likely to be anxiety-inducing. Given that this anxiety is aversive, there will be a strong temptation to avoid thinking about, let alone planning for, retirement until it is too close to avoid. “Out of mind, out of sight” may increase utility today, but at the cost of reducing welfare tomorrow.17

VI. OTHER APPLICATIONS

V.A. Anticipatory Pleasure, Savoring, and Planning

While our discussion has focused on anxiety, not all anticipatory emotions are aversive. Loewenstein [1987] found that there was a general preference for delaying a kiss from a romantic ideal. This is suggestive of a savoring motive, whereby subjects prefer to extend the period in which they can savor the anticipation of an enjoyable prize. According to our theory, this savoring motive should give rise to time inconsistency, and a resulting desire to use commitment devices to overcome the time inconsis-

16. Bernheim, Skinner, and Weinberg [1997] show that many households go through significant reductions in food consumption upon retirement, as if their retirement came by surprise, and caused a large reduction in perceived wealth.

17. A related issue concerns the optimal supply of financial education. Bernheim and Garrett [1996] have shown that savings and participation in financial education counseling are strongly positively correlated. It may well be that encouraging, or even “forcing” more households to participate in these programs would result in them being better prepared for retirement. But would there be a cost in terms of increased anxiety in the preretirement period?
tency. After all, if you currently prefer to delay an experience in order to savor it, will you not feel the same way tomorrow?

Caplin and Leahy [1997] provide another example of savoring which illuminates the commitment motive. We consider a family contemplating a future vacation to one of two locations. The family is indifferent between the two locations given the information that they initially possess. The twist is that they enjoy thinking about the impending trip, and find it is easier to anticipate the vacation if they know which vacation they will take. This means that the household’s preferences over possible vacation lotteries will change over time. In order to facilitate anticipation, they would like to know their destination some time prior to departure. At the point of departure, however, all that matters is which vacation looks more promising at that date, which may or may not coincide with the choice that they would have made earlier. Given this potential change in attitude, we show that the family may wish to commit to one of the locations at an early stage by getting a nonrefundable airline ticket.

V.B. Suspense and Gambling

Another positive anticipatory emotion is suspense. In Caplin and Leahy [1997] we define suspense as the pleasure experienced immediately prior to the anticipated resolution of uncertainty, and posit that it is positively related (up to a point) to the amount that is at stake on the outcome of an event. This provides a simple reason for agents to bet that their emotional favorite will win in a sporting event. By betting on their favorite, agents increase their stake in the outcome, thereby heightening feelings of suspense. Our prediction that agents will prefer to bet on their emotional favorites finds strong support in the empirical analysis of Babad and Katz [1991], who study betting on soccer matches in Israel.

V.C. Moods and Risk-Taking Behavior

It has long been conjectured that personal mood impacts risk-taking behavior, a factor that is per force ignored in the standard theory in which the individual’s attitude to risk is summarized by a fixed function describing the expected utility of wealth. Isen [1993] notes one important effect. Her experimental research indicates that in many settings "feeling good" gives rise to a higher degree of risk aversion, in the sense that the individ-
ual in a good mood rejects as too risky gambles that are acceptable to subjects in a bad mood.

Going beyond the experiments, Isen [1993] has presented a theoretical explanation that fits in squarely with our model of rational behavior: “The relative risk aversion observed in positive affect subjects considering real risks can be interpreted in terms of affect maintenance. People who are feeling happy risk losing that state, as well as any tangible stake, if they lose a gamble. Therefore, with more to lose than controls, they are more risk-averse than controls” [p. 270]. It is straightforward to use our model of mental states to rationalize precisely this pattern of behavior, by making the second-period mental state depend upon the first-period mental state. More broadly, our model offers hope for analyzing the role of moods not only in individual decision making, but also in strategic settings. Are there times during the bargaining process when it would make sense to try to influence the mood of your opposite number?

VI. Comments on the Literature

VI.A. Why Retain the Substitution Axiom?

Our approach to analyzing psychological aspects of decisions is to retain the classical assumption of rationality in its full force. We encode the anticipatory emotions as part of the prize space, and apply the standard substitution axiom on this richer domain. Pope [1985] provides an alternative vision of anticipation and the substitution axiom. She believes that the introduction of time invalidates the substitution axiom, while we believe that it restores it (provided that one uses the appropriate state space).

According to Pope [1985], when one allows for feelings in the period prior to the realization of a lottery, the intuitive argument in favor of the substitution axiom may be false. Lottery A may be preferred to lottery B, but these lotteries may give rise to entirely different anticipatory responses when mixed with lottery C.

We agree with Pope that anticipatory feelings invalidate the standard static version of the substitution axiom. Rather than abandon the axiom, we choose instead to encode the prior feelings

19. In this sense we lie within the tradition summarized in Section 2 of Rabin’s [1998] recent survey article on economics and psychology.
as prizes in the lottery. We believe that when the substitution axiom is applied on this larger psychological state space, the axiom regains its original appeal. Rejection of the substitution axiom is valid only if one believes that there can be no complete model of psychological states, as when one is forced to introduce “anticipatory feelings about anticipatory feelings about . . . ,” and so on ad infinitum.

VI.B. PEU and Nonexpected Utility Theory

In contrast with our approach, much recent progress in the theory of choice under uncertainty has involved the rejection of the substitution axiom for lotteries, and the development of various nonexpected utility theories. Many of these theories have been developed in response to the observed violations of the substitution axiom, such as the Allais paradox. In fact, the chief goal of many nonexpected utility theories is to relax the substitution axiom as little as possible, while nevertheless allowing for some limited class of violations.

One important advantage of our formulation over static nonexpected utility models is that the latter theories attempt to telescope a dynamic pattern of feelings into a single static utility function. Many subtle phenomena in the psychology of risk-taking require explicitly dynamic formulations. For example, while the models of disappointment due to Bell [1985] and Loomes and Sugden [1986] are explicit concerning the way in which a given lottery produces feelings of disappointment, they remain static. Among other things, this means that the models do not include an anticipatory phase. Have you ever felt disappointed about an outcome without having experienced prior feelings of hopefulness?

The model that is most directly related to ours is the dynamic nonexpected utility model of Kreps and Porteus [1978]. This was the first model to allow agents to have a preference for delaying the receipt of information (see Grant, Kajii, and Polak [1998] for an exploration of this aspect of the model). In fact, our model can be seen in its reduced form as a generalization of their model, in

20. In this respect, at least one prominent nonexpected utility theorist agrees. Machina wrote: “For my part, I will grant that separability may be rational provided the descriptions of consequences are sufficiently deep to incorporate any relevant emotional states, such as disappointment (e.g., at having won $0 when you might have won $5 million), regret . . . and so on” [Machina 1989, p. 1662].

21. Psychological recognition of the link between these two emotions can be found in the very interesting book by Ortony, Clore, and Collins [1988].
which we relax their assumption that preferences are time consistent. This is an important generalization, since time consistency appears to be a very unnatural assumption in cases with anticipatory emotions, as we have seen.

Caplin and Leahy [1999] provide further motivation for an expected utility approach such as ours rather than a nonexpected utility approach such as that of Kreps and Porteus. We model a doctor’s decision on whether or not to provide a patient with detailed information about an upcoming operation. We show that the Kreps-Porteus model is not sufficiently rich to model the doctor’s decision. The subtle point is that the doctor must be able to decide what to do in the face of a superior understanding of the lottery that the patient is about to face. The decision on whether or not to provide information at this stage must be based on an assessment of whether or not to leave the patient with an illusion about tomorrow’s lottery. The Kreps-Porteus model is not rich enough to cover preferences over illusions, while such preferences can be straightforwardly assessed in our psychological expected utility theory.

VII. SOME POTENTIAL ISSUES FOR POLICY

Anticipatory emotions such as anxiety are a function of agents’ beliefs about the future. This suggests that policy makers might want to consider how policies affect beliefs, especially policies that involve the provision of information. For example, suppose that the Federal Reserve possesses a piece of bad news such as the imminent failure of a large bank, and that the Fed knows that in all likelihood it will be able to solve the problem before the problem gets out of hand and affects other financial institutions or the economy as a whole. Should the Fed release this information immediately, or should it hold off until the trouble is past? Barring public misunderstanding of the news, if agents are expected utility maximizers, then the Fed should provide the news. The planning advantages of more information argue for immediate disclosure of any information relevant to decision making.

In a model with anxiety, however, the situation is more complex. The Fed must weigh the planning advantages of superior information against the possibility that an announcement would needlessly cause investors to worry, or worse to panic. If the Fed believes that it has the situation under control, the optimal policy would appear to be to suppress the information.
The lost planning advantages would be small, and since anxiety depends on possibility and not probability, the potential reduction in anxiety is large.

Policy makers might also be concerned with the actions agents take to affect their own beliefs. Consider an example from the medical arena. If agents avoid information that makes them anxious, they might be reluctant to go to the doctor or test themselves for diseases such as breast cancer. Such avoidant behavior has clear implications for health and health-care costs. In response to the apparent pattern of avoidant behavior, there has recently been increased attention paid to new formats for supplying medical advice and medical information to the general population. A number of studies indicate that “psychologically-appropriate” forms of information-intervention can improve health-care outcomes and lower costs on the order of 20 percent [Fries 1998].

VIII. CONCLUDING REMARKS

In this paper we have introduced the psychological expected utility model, and used it to analyze the impact on decision making of anticipatory feelings. The broader goal of our research agenda is to open up a variety of psychologically interesting phenomena to rational analysis, and in this respect our work has just begun.

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